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A gamma type distribution involving a confluent hypergeometric function of the second kind*

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Abstract

In the present work a new gamma type distribution is obtained which involves a confluent hypergeometric function of the second kind. A generalized form of the incomplete gamma function and its complementary are introduced to obtain some statistical functions. The associated statistical functions with the probability density function are deduced such as the k-moment, expect value, risk function, half-life function and other special cases.

Key words: Generalized distribution, incomplete gamma type, hypergeometric confluent function of the second kind.

Una distribución generalizada tipo gamma que involucra la función hipergeométrica confluente de segunda clase

Resumen

En el presente trabajo se obtiene una nueva distribución tipo gamma que involucra a la función hipergeométrica confluente de segunda clase. Una forma generalizada de la función gamma incompleta y su forma complementaria son introducidas para obtener algunas funciones estadísticas. Se deducen las funciones estadísticas asociadas con la función de densidad de probabilidad, tales como el k-ésimo momento, el valor esperado, la función de riesgo y la función de vida media, y otros casos especiales.

Palabras clave: Distribución generalizada, tipo gamma incompleta, función hipergeométrica confluente de segunda clase.

Introduction

A new class of functions were introduced and developed by Virchenko [1, 2], which may be called τ-hypergeometric and τ-confluent hypergeometric functions. Those functions are natural generalizations of classical hypergeometric functions. Agarwal and Kalla [3] have defined a generalized gamma function distribution derived from a generalized Kobayashi gamma function [4], which is a confluent hypergeometric function of the second kind.

A unified form of gamma type distribution, was given by Kalla and others [5] based on the generalized gamma function defined by Al-Musallan and Kalla [6, 7].

Recently A. Al-Zamel [8] has introduced a new gamma type distribution involving the τ-confluent hypergeometric function and has dis-

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cussed some basic functions associated with the distribution.

In this paper, we present first some properties of τ-hypergeometric function and the hypergeometric confluent function of the second kind and we also define a generalized form of the incomplete gamma function and its complementary. Moreover a density function asociated with the hypergeometric function of second kind is defined. The gamma, generalized gamma, Weibull and another type of gamma incomplete distribution are obtained as particular cases of the density generalized function. Some properties associated with the density function and other frequently used functions such as the *k-th* moment, risk function, life time are derived.

The gamma function is defined [9], as follows:

$$\Gamma(a) = \int_0^\infty e^{-t} t^{a-1} dt, \text{ where } \mathrm{Re}(a) > 0. \quad (1)$$

The incomplete gamma function and the complementary gamma function are defined as follows:

$$\gamma(a, x) = \int_0^x e^{-t} t^{a-1} dt, \text{ where } \mathrm{Re}(a) > 0, \quad (2)$$

$$\Gamma(a, x) = \int_0^\infty e^{-t} t^{a-1} dt, \text{ where } \mathrm{Re}(a) > 0 \quad (3)$$

A special case of the function ${}_p\Psi_q$ was given by Galué [10]. This function is defined by

$${}_p\Psi_q\left[\begin{matrix}(A_1,1),\ldots,(A_p,1);\\(B_1,1),\ldots,(A_q,1);\end{matrix}\, x\right] = \frac{\prod_{j=1}^{p}\Gamma(A_j)}{\prod_{j=1}^{q}\Gamma(B_j)}\, {}_pF_q\left[\begin{matrix}A_1,\ldots,A_p;\\B_1,\ldots,B_q;\end{matrix}\, x\right]. \quad (4)$$

The confluent hypergeometric is defined [9] as follows:

$${}_1\Phi_1(a;c;x) = M(a;c;x) = \sum_{n=0}^{\infty} \frac{(a)_n}{(c)_n}\frac{x^n}{n!}$$
$$\text{with } |x| < \infty;\ c \neq 0, -1, -2, \ldots \quad (5)$$

where $(\lambda)_n = \dfrac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \lambda(\lambda+1)\ldots(\lambda+n-1)$ is the Pochhammer symbol.

The confluent hypergeometric function of second kind is defined by [9]

$$(\lambda)_n = \frac{\Gamma(\lambda+n)}{\Gamma(\lambda)} = \lambda(\lambda+1)\ldots(\lambda+n-1)$$

$$U(a;c;x) = \frac{\pi}{\mathrm{sen}\,(\pi c)}\left[\frac{M(a;c;x)}{\Gamma(1+a-c)\Gamma(c)} - \frac{x^{1-c}M(1+a-c;2-c;x)}{\Gamma(a)\Gamma(2-c)}\right] \quad (6)$$

The integral representation of $U(a;c;x)$ is given by [9]

$$U(a;c;x) = \frac{1}{\Gamma(a)}\int_0^\infty e^{-xt} t^{a-1}(1+t)^{c-a-1} dt,$$
$$a > 0,\ c > 0,\ c \neq 1, 2, \ldots \quad (7)$$

The function $U(a;c;x)$ satisfied the following recurrency relations and differential formulas

$$U(a;c;x) - aU(a+1;c;x) = U(a;c-1;x), \quad (8)$$

$$(c-a)U(a;c;x) + U(a-1;c;x) = xU(a;c+1;x) \quad (9)$$

Generalized gamma function

By using the confluent hypergeometric function of the second kind given in (6), we define the following incomplete generalized gamma function as:

$${}_\lambda\gamma_w(p,\delta;a;c;v) = \int_0^w x^{\lambda-1} e^{-px^\delta} U(a;c;vx^\delta)dx \quad (10)$$

where $x > 0$, $\delta > 0$, $p > 0$, a and c are arbitrary constants.

In the same way the complementary generalized gamma function is given by:

$${}_\lambda\Gamma_w(p,\delta;a;c;v) = \int_w^\infty x^{\lambda-1} e^{-px^\delta} U(a;c;vx^\delta)dx, \quad (11)$$

where $x > 0$, $\delta > 0$, $p > 0$, a and c are arbitrary constants.

Some particular cases of the equations (10) and (11) are

$${}_{\lambda}\gamma_{w}(1,1;1;2;v) = \frac{1}{v}\int_{0}^{w} x^{\lambda-2}e^{-x}dx = \frac{1}{v}\gamma(\lambda-1, w). \quad (12)$$

$${}_{\lambda}\gamma_{w}(p,\delta;1;2;v) = \frac{1}{v}\int_{0}^{w} x^{\lambda-\delta-1}e^{-px^{\delta}}dx =$$

$$\frac{1}{v\delta p^{\frac{\lambda}{\delta}-1}}\gamma\left(\frac{\lambda}{\delta}-1, pw^{\delta}\right). \quad (13)$$

$${}_{\lambda}\Gamma_{w}(1,1;1;2;v) = \frac{1}{v}\int_{w}^{\infty} x^{\lambda-1}e^{-x}dx = \frac{1}{v}\Gamma(\lambda-1, w). \quad (14)$$

$${}_{\lambda}\Gamma_{w}(p,\delta;1;2;v) = \frac{1}{v}\int_{w}^{\infty} x^{\lambda-\delta-1}e^{-px^{\delta}}dx = \frac{1}{v\delta p^{\frac{\lambda}{\delta}-1}} \times$$

$$\Gamma\left(\frac{\lambda}{\delta}-1, pw^{\delta}\right). \quad (15)$$

The generalized incomplete gamma function can be written as a classic series of the incomplete gamma function. Using (10), (6) and (5), then solving the integrals we obtain

$${}_{\lambda}\gamma_{w}(p,\delta;a;c;v) =$$

$$\frac{\pi}{\delta p^{\frac{\lambda}{\delta}}\mathrm{sen}\,(\pi c)}\left[\frac{\sum_{n=0}^{\infty}\frac{(a)_n}{(c)_n},\frac{(v/p)^n}{n!}}{\Gamma(1+a-c)\Gamma(c)}\gamma\left(\frac{\lambda}{\delta}+n, pw^{\delta}\right) - \right.$$

$$\left.\frac{1}{p^{\frac{1-c}{\delta}}}\frac{\sum_{n=0}^{\infty}\frac{(1+a-c)_n}{(2-c)_n},\frac{(v/p)^n}{n!}}{\Gamma(1+a-c)\Gamma(c)}\gamma\left(\frac{\lambda-c+1}{\delta}+n, pw^{\delta}\right)\right]. \quad (16)$$

We obtain the recurrence formulas to the generalized incomplete gamma function and the generalized complementary gamma function using the relation (8) and (9),

$${}_{\lambda}\gamma_{w}(p,\delta;a;c;v) = {}_{\lambda}\gamma_{w}(p,\delta;a;c-1;v) + a\ {}_{\lambda}\gamma_{w}(p,\delta;a+1;c;v). \quad (17)$$

$${}_{\lambda}\Gamma_{w}(p,\delta;a-1;c;v) = {}_{\lambda+1}\Gamma_{w}(p,\delta;a;c+1;v) + (a-c)\ {}_{\lambda}\Gamma_{w}(p,\delta;a;c;v). \quad (18)$$

Probability density function

In this case, we use the confluent hypergeometric function of the second kind to define the probability density function as follows (see equation (19) below) (Figure 1), where λ, δ, $p > 0$, λ, v, p are constants such as $0 < v < p$, $c < 1$, $c \notin Z$ and a, $1+a-c \notin Z^{-}$.

$$A(p,\delta,\lambda,a,c,v) = p^{1-c}\Gamma(a)\Gamma\left(\frac{\lambda}{\delta}\right)\Gamma(2-c)\times$$

$${}_{2}F_{1}\left(a,\frac{\lambda}{\delta};c;\frac{v}{p}\right) \quad (20)$$

$$B(p,\delta,\lambda,a,c,v) = v^{1-c}\Gamma(1+a-c)\Gamma(c)\Gamma\left(\frac{\lambda}{\delta}-c+1\right)\times$$

$${}_{2}F_{1}\left(1+a-c,\frac{\lambda}{\delta}-c+1;2-c;\frac{v}{p}\right). \quad (21)$$

We conclude that

$$\int_{0}^{\infty} x^{\lambda-1}e^{-px^{\delta}}U(a;c;vx^{\delta})dx = \frac{\pi}{\delta\,\mathrm{sen}\,(\pi c)p^{\frac{\lambda}{\delta}}}\times$$

$$\left[\frac{A(p,\delta,\lambda,a,c,v)-B(p,\delta,\lambda,a,c,v)}{p^{1-c}\Gamma(1+a-c)\Gamma(c)\Gamma(a)\Gamma(2-c)}\right]. \quad (22)$$

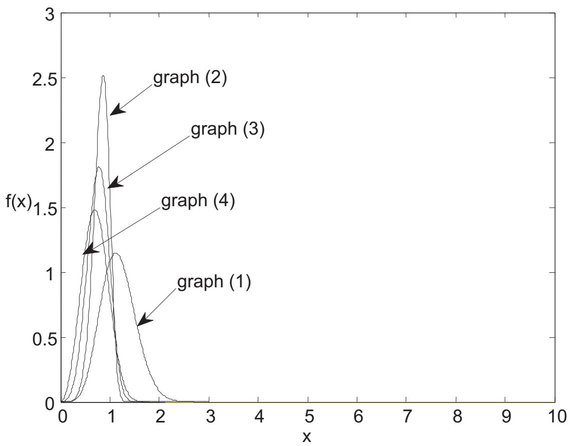
Another way to represent (22) is using the equation (4) of the ${}_{p}\Psi_{q}(x)$, which gives:

$$\int_{0}^{\infty} x^{\lambda-1}e^{-px^{\delta}}U(a;c;vx^{\delta})dx = \frac{\pi}{\delta\,\mathrm{sen}\,(\pi c)p^{\frac{\lambda}{\delta}}}\times$$

$$\left[\frac{A'(p,\delta,\lambda,a,c,v)-B'(p,\delta,\lambda,a,c,v)}{p^{1-c}\Gamma(1+a-c)\Gamma(c)\Gamma(a)\Gamma(2-c)}\right]$$

where

$$f(x) = \frac{\delta\,\mathrm{sen}\,(\pi c)p^{\frac{\lambda}{\delta}-c+1}\Gamma(1+a-c)\Gamma(c)\Gamma(a)\Gamma(2-c)x^{\lambda-1}e^{-px^{\delta}}U(a;c;vx^{\delta})}{\pi\left[A(p,\delta,\lambda,a,c,v)-B(p,\delta,\lambda,a,c,v)\right]};\ x \geq 0 \qquad f(x) = 0;\ x < 0. \quad (19)$$



graph (1) for $p = 2, \delta = 2, \lambda = 6, a = 3, c = 0.31, v = 0$;
graph (2) for $p = 2, \delta = 6, \lambda = 6, a = 3, c = 0.31, v = 0$;
graph (3) for $p = 2, \delta = 4, \lambda = 4, a = 3, c = 0.31, v = 0$;
graph (4) for $p = 2, \delta = 3, \lambda = 3, a = 3, c = 0.31, v = 0$.

Figure 1. Representation of $f(x)$ for values of the parameters.

$$A'(p,\delta,\lambda,a,c,\upsilon) = p^{1-c}\Gamma(c)\Gamma(2-c)\ {}_2\Psi_1\left[\begin{matrix}(a,1),\left(\frac{\lambda}{\delta},1\right); & \frac{\upsilon}{p}\\ (c,1); & \end{matrix}\right], \tag{23}$$

$$B'(p,\delta,\lambda,a,c,\upsilon) = \upsilon^{1-c}\Gamma(c)\Gamma(2-c)\ {}_2\Psi_1\left[\begin{matrix}(1+a-c,1),\left(\frac{\lambda}{\delta}-c+1,1\right); & \frac{\upsilon}{p}\\ (2-c,1); & \end{matrix}\right]. \tag{24}$$

It follows that

$$\int_0^\infty f(x)dx = 1$$

and therefore $f(x)$ is a probability density function [19].

Furthermore, it is inmediately that

i) $\lim\limits_{x\to 0^+} f(0) = 0$, if $\lambda > 1$.

ii) Setting $x = 0$

$$f(0) = \frac{\delta\Gamma(a)\Gamma(2-c)p^{\frac{\lambda}{\delta}+1-c}}{A(p,\delta,\lambda,a,c,\upsilon) - B(p,\delta,\lambda,a,c,\upsilon)} \quad \text{if } \lambda = 1.$$

iii) $\lim\limits_{x\to 0^+} f(x) = \infty$, such that $\lambda < 1$.

iv) $\lim\limits_{x\to\infty} f(x) = 0$.

Some special cases of the density function $f(x)$, are:

1) Setting $\upsilon = \delta = 1$ in the equation (19), (see equation (25) below).

2) If $\upsilon = 0$ in the equation (19), gives:

$$f(x) = \frac{\delta p^{\frac{\lambda}{\delta}}}{\Gamma\left(\frac{\lambda}{\delta}\right)} x^{\lambda-1}e^{-px^{\delta}}, \tag{26}$$

which (26) may be called the generalized Weibull density function.

3) Let $\delta = \lambda$ in (26), we get the following form of $f(x)$:

$$f(x) = \lambda p x^{\lambda-1}e^{-px^{\lambda}}. \tag{27}$$

4) If $\delta = 1$, in (26),

$$f(x) = \frac{p^{\lambda}x^{\lambda-1}e^{-px}}{\Gamma(\lambda)},\ x > 0;\ p > \lambda > 0. \tag{28}$$

Statistical functions

In this section we present the basic statistical functions associated with the density function defined in (19).

k-th moment

μ'_k is the *k-th* moment about the origin of the continuous real random variable x (for simplicity beginning from this point we change the notation X by x) with density function $f(x)$ given by:

$$f(x) = \frac{\text{sen}(\pi c)p^{\lambda-c+1}\Gamma(1+a-c)\Gamma(c)\Gamma(a)\Gamma(2-c)x^{\lambda-1}e^{-px^{\delta}}u(a;c;x)}{\pi\left[A(p,1,\lambda,a,c,1) - B(p,1,\lambda,a,c,1)\right]}. \tag{25}$$

$$\mu'_k = \int_0^\infty x^k f(x)dx \quad (29)$$

Using $f(x)$ given in (19), with the conditions stated for the convergence, we find from the density function defined in (19) that the k-th moment is

$$\mu'_k = \frac{C(p,\delta,\lambda+k,a,c,\upsilon) - D(p,\delta,\lambda+k,a,c,\upsilon)}{p^{\frac{k}{\delta}}\left[A(p,\delta,\lambda,a,c,\upsilon) - B(p,\delta,\lambda,a,c,\upsilon)\right]} \quad (30)$$

where A and B are given by the relations (20) and (21), respectively

$$C(p,\delta,\lambda+k,a,c,\upsilon) = p^{1-c}\Gamma(a)\Gamma\left(\frac{\lambda+k}{\delta}\right)\Gamma(2-c)\ {}_2F_1\left(a,\frac{\lambda+k}{\delta};c;\frac{\upsilon}{p}\right) \quad (31)$$

$$D(p,\delta,\lambda+k,a,c,\upsilon) = \upsilon^{1-c}\Gamma(1+a-c)\Gamma(c)\Gamma\left(\frac{\lambda+k}{\delta}-c+1\right)\times {}_2F_1\left(1+a-c,\frac{\lambda+k}{\delta}-c+1;2-c;\frac{\upsilon}{p}\right). \quad (32)$$

Putting $k=1$ in (30) gives the mean:

$$\mu'_k = E'[x] = \frac{C(p,\delta,\lambda+1,a,c,\upsilon) - D(p,\delta,\lambda+1,a,c,\upsilon)}{p^{\frac{c}{\delta}}\left[A(p,\delta,\lambda,a,c,\upsilon) - B(p,\delta,\lambda,a,c,\upsilon)\right]}.$$

Expected value

Let $\Psi(x)$ be a function of a continuous randon variable x with the density function $f(x)$ of (19), the expected value $\Psi(x)$ is denoted by $E[\Psi(x)]$

$$E[\Psi(x)] = \int_0^\infty \Psi(x)f(x)dx \quad (33)$$

where $f(x)$ is the density function defined in (19).

The existence of expected value is related to and depend on the functions in the predetemined interval. In this case it can also be interpreted as an integral transform of the density function $f(x)$ with respect to the kernel $\Psi(x)$.

As an example, we can make use of Mellin transform assuming that x is a positive real random variable with the density function $f(x)$, $x \geq 0$ ($f(x)=0$ for $x>0$), then

$$E[x^{s-1}] = \int_0^\infty x^{s-1}f(x)dx = M\left[f(x);s\right], \quad (\alpha < \mathrm{Re}(s) < \beta) \quad (34)$$

where $M[f(x);s]$ is the Mellin transform of $f(x)$.

Now, let $\Psi(x) = x^{s-1}$ and $f(x)$ the density function, then the expected value is

$$E[x^{s-1}] = \frac{E(p,\delta,\lambda+s-1,a,c,\upsilon) - F(p,\delta,\lambda+s-1,a,c,\upsilon)}{p^{\frac{s-1}{\delta}}\left[A(p,\delta,\lambda,a,c,\upsilon) - B(p,\delta,\lambda,a,c,\upsilon)\right]}, \quad (0 < \mathrm{Re}(s) < 1). \quad (35)$$

where

$$E(p,\delta,\lambda+s-1,a,c,\upsilon) = p^{1-c}\Gamma(a)\Gamma\left(\frac{\lambda+s-1}{\delta}\right)\Gamma(2-c)\ {}_2F_1\left(a,\frac{\lambda+s-1}{\delta};c;\frac{\upsilon}{p}\right)$$

$$F(p,\delta,\lambda+s-1,a,c,\upsilon) = \upsilon^{1-c}\Gamma(1+a-c)\Gamma\left(\frac{\lambda+s-1}{\delta}-c+1\right)\Gamma(c)\times {}_2F_1\left(1+a-c,\frac{\lambda+s-1}{\delta}-c+1;2-c;\frac{\upsilon}{p}\right).$$

A and B are given by the relations (20) and (21), respectively.

The moment generating function is given by

$$E[e^{xt}] = M_x(t) = \int_0^\infty e^{xt} f(x)dx = K\int_0^\infty x^{\lambda-1}e^{-px^\delta+xt}U(a;c;\upsilon x^\delta)dx \quad (36)$$

where

$p > t \geq 0$ and

$$K = \frac{\delta \operatorname{sen}(\pi c)p^{\frac{\lambda}{\delta}-c+1}\Gamma(1+a-c)\Gamma(c)\Gamma(a)\Gamma(2-c)}{\pi\left[A(p,\delta,\lambda,a,c,\upsilon) - B(p,\delta,\lambda,a,c,\upsilon)\right]}$$

Setting $\delta = 1$ in (36), we have

$$E[e^{xt}] = M_x(t) = k\int_0^\infty x^{\lambda-1}e^{-(p-t)x}u(a;c;vx)dx. \quad (37)$$

Using the expression (22), solving the integral of (37) and replacing K, it gives

$$E[e^{xt}] = M_x(t) = \frac{p^{\lambda-c+1}\left[G(p-t,1,\lambda,a,c,v) - H(p-t,1,\lambda,a,c,v)\right]}{(p-t)^{\lambda-c+1}\left[A(p,1,\lambda,a,c,v) - B(p,1,\lambda,a,c,v)\right]} \quad (38)$$

where

$$G(p-t,1,\lambda,a,c,v) = (p-t)^{1-c}\,\Gamma(\lambda)\Gamma(a)\Gamma(2-c)\,{}_2F_1\left(a,\lambda;c;\frac{v}{p-t}\right),$$

$$H(p-t,1,\lambda,a,c,v) = v^{1-c}\Gamma(\lambda-c+1)\Gamma(1+a-c)\Gamma(c)\,{}_2F_1\left(1+a-c,\lambda-c+1;2-c;\frac{v}{p-t}\right)$$

and where A and B are given in (20) and (21), respectively.

Risk function

The risk function (failure rate) $h(x)$ is given by

$$h(x) = \frac{f(x)}{s(x)},\ s(x) \neq 0,\ x > 0 \quad (39)$$

where $s(x)$ is the survival o realibility function defined as

$$s(x) = 1 - F(x),\ x > 0 \quad (40)$$

and $F(x)$ is the cumulative density function, which is defined through the formula

$$F(x) = \int_0^x f(t)dt. \quad (41)$$

It is obvious that the survival function is in this case (see equation (42) below)

A particular case results when $v = 0$ in the risk function which is obtained from the density $f(x)$ of (19)

$$h(x) = \frac{\delta p^{\frac{\lambda}{\delta}} x^{\lambda-1}e^{-px^\delta}}{\Gamma\left(\frac{\lambda}{\delta}, px^\delta\right)}. \quad (43)$$

Setting, $\delta = p = 1$ we obtain the corresponding result for the density gamma function

$$h(x) = \frac{x^{\lambda-1}e^{-px^\delta}}{\Gamma(\lambda, x)}. \quad (44)$$

The mean life function

For a continuous random real variable x with density function $f(x)$ the mean life function is defined as

$$k(x) = \frac{1}{s(x)}\int_x^\infty (y-x)f(y)dy. \quad (45)$$

Replacing (42) in (45) and using the results of the last integrals we get

$$K(x) = \frac{{}_{\lambda+1}\Gamma_x(p,\delta;a;c;v)}{{}_{\lambda}\Gamma_x(p,\delta;a;c;v)} - x. \quad (46)$$

For $v = 0$ we obtain the following mean life function of the generalized density Weibull function in (26). It then follows that:

$$K(x) = \frac{\Gamma\left(\frac{\lambda+1}{\delta}, px^\delta\right)}{p^{\frac{1}{\delta}}\Gamma\left(\frac{\lambda}{\delta}, px^\delta\right)} - x. \quad (47)$$

$$s(x) = \frac{\delta \operatorname{sen}(\pi c) p^{\frac{\lambda}{\delta}-c+1}\Gamma(1+a-c)\Gamma(c)\Gamma(a)\Gamma(2-c)\,{}_{\lambda}\Gamma_x(p,\delta;a;c;v)}{\pi\left[A(p,\delta,\lambda,a,c,v) - B(p,\delta,\lambda,a,c,v)\right]}. \quad (42)$$

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