Rev. Téc. Ing. Univ. Zulia. Vol. 18, No. 3, 305-311, 1995

Stability of an Oldroydian fluid layer in a horizontal magnetic field

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Abstract

The effects of Hall currents on the Rayleigh-Taylor instability of an incompressible, finitely conducting viscoelastic fluid were investigated. It is assumed that the fluid is permeated by a uniform two-dimensional horizontal magnetic field. It is shown that the solution is characterized by a variational principle. Proper solutions were obtained for a semi-infinite fluid in which the density varies exponentially in the vertical direction. The dispersion relation was derived and solved numerically. It was found that Hall currents and finite conductivity have a destabilizing influence, while viscosity and elasticity have a stabilizing influence on the growth rate of the unstable mode of disturbances.

Key words: Hall currents, stability, finite conductivity, viscosity, elasticity.

Estabilidad de una capa del fluido oldroydiano en un campo magnético horizontal

Resumen

En el presente trabajo se investigaron los efectos de las corrientes de Hall sobre la inestabilidad Rayleigh-Taylor de un fluido conductor finito, incompresible y viscoelástico. Se asumió que el fluido está permeado por un campo magnético uniforme bidimensional y horizontal. Se demostró que la solución está caracterizada por un principio variacional. Se obtuvieron soluciones apropiadas para un fluido semi-infinito en el cual la densidad varia exponencialmente en la dirección vertical. La relación de dispersión se derivó y se solucionó numéricamente. Se halló que las corrientes de Hall y la conductividad finita tienen una influencia desestabilizadora, mientras que la viscosidad y la elasticidad tienen una influencia estabilizadora en la tasa de crecimiento del modo inestable de las perturbaciones.

Palabras claves: Corrientes de Hall, estabilidad, conductividad finita, viscosidad, elasticidad.

Introduction

The study of the equilibrium of an incompressible inviscid fluid of variable density was first undertaken by Rayleigh [1]. Taylor [2] studied the stability problem of a heterogeneous fluid accelerated in the direction perpendicular to the plane of stratification. Since then several authors have investigated the Rayleigh-Taylor instability problem under various physical assumptions. A comprehensive account of these investigations carried out under various assumptions of hydrodynamics and hydromagnetics has been given by Chandrasekhar [3].

Several authors (e.g. Kruskal and Schwarzschild [4], Hide [5]) have pointed out the stabilizing character of the magnetic field on this stability problem. The effects of Hall currents are of considerable importance in the dynamics of interstellar matter and several other physical situations. Several authors (e.g. Hosking [6], Singh and Tandon [7], Ariel [8], Bhowmik [9], Bhatia [10]) have studied the effects of Hall currents on the Rayleigh-Taylor instability problem in hydromagnetic and found in general that the Hall effect is destabilizing and gives rise to news unstable modes.

The effects of viscosity are also of considerable importance in astrophysical situations. Vest and Arpaci [11] have considered the stability of a horizontal layer of a viscoelastic fluid heated from below and obtained the conditions under which a thermally induced overstability occurs in a Maxwellian fluid. Bathia and Steiner [12] have studied the thermal instability of a Maxwell fluid in hydro-magnetic. Sharma [13] studied the Rayleigh-Taylor instability of a viscoelastic fluid through a porous medium. More recently, Samria, Reddy and Prasad [14] have investigated the MHD flow of an elastoviscous fluid past a porous flat plate.

Gupta and Bhatia [15] have studied the inestability of superposed, partially ionized plasmas in a two-dimensional horizontal magnetic field. It is, therefore, of importance to examine the effects of Hall currents and magnetic resistivity on the Rayleigh-Taylor instability of a viscoelastic fluid. This aspect forms the basis of this paper. For an ideally conducting viscoelastic fluid, this problem was recently studied by Ali [16].

Perturbation Equations

We consider the motion of an incompressible, finitely conducting, viscoelastic fluid in the presence of a uniform magnetic field.

The essential difference between Newtonian fluid and non-Newtonian fluids is that while Newtonian or Stokesian fluid are characterized by a linear relation between the stress tensor and the rate of strain tensor, the non-Newtonian fluids are characterized by a non-linear relationship between the stress tensor and the rate of strain tensor. Broadly speaking non-Newtonian fluids can be divided into viscoinelastic fluids, viscoelastic fluids, polar fluids, dipolar fluids, anisotropic fluids, fluids with microstructure, and heat conducting nematic liquid crystals. For a Newtonian fluid the problem of the stability of a semi-infinite layer in a horizontal one-dimensional magnetic field (including also the effects of neutral gas friction was studied earlier by the first author Bathia [10]). The aim of this paper is to study the problem of stability of a horizontal layer of a non-Newtonian fluid of variable density. We study here this problem for an Oldroydian viscoelastic fluid for which the costitutive equation is

$$(1+\lambda\frac{\partial}{\partial t})\tau_{ij}=2\mu(1+\lambda_o\frac{\partial}{\partial t})e_{ij}\ , \tag{1}$$

where τ_{ij} is the viscous stress tensor, μ is coefficient of viscosity, λ and $\lambda_o(\lambda_o < \lambda)$ are respectively the stress relaxation and strain retardation time, and

$$e_{ij}=\frac{1}{2}(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i})\ , \tag{2}$$

is the rate of strain tensor, Here u_i is velocity. The viscous stress tensor τ_{ij} is related to the total stress tensor T_{ij} through

$$T_{ij}=-p\delta_{ij}+\tau_{ij}\ , \tag{3}$$

where δ_{ij} is kronecker tensor and p is the scalar pressure.

The equation of motion of an electrically conducting fluid moving in a uniform magnetic field.

$$\rho\frac{Dv_i}{D_t}=-g\rho\lambda_i+\frac{\partial}{\partial x_j}T_{ij}+\mu_e\varepsilon_{ijk}\varepsilon_{jlm}\frac{\partial H_{ln}}{\partial x_j}H_k \tag{4}$$

where

$$\frac{D}{Dt}=\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x_i}$$

is the mobile operator, H_1 is magnetic field, ρ is density of the fluid, g is gravity and $\lambda_i=(0,0,1)$ is a unit vector along the vertical. For an Oldroydian viscoelastic fluid, the hydromagnetic equation of motion (4) becomes, on using the constitutive equation (3) in conjunction with (1),

$$\rho(1+\lambda\frac{\partial}{\partial t})\frac{D\vec{u}}{Dt}=(1+\lambda\frac{\partial}{\partial t})\,[-\nabla p+\mu_e(\nabla x\vec{H})\,x\,\vec{H}+\vec{g}\rho]$$
$$+(1+\lambda_o\frac{\partial}{\partial t})\,[\mu\nabla^2\vec{u}+(\nabla\vec{u})\cdot\nabla\mu+(\nabla\mu)\cdot\nabla\vec{u}]\ , \tag{5}$$

where $\vec{g}=(0,0,-g)$. The relavant equations of motion of an incompressible viscoelastic Oldroydian fluid in the presence of the effects of Hall currents and magnetic resistivity are, therefore, equation (5) and

$$\frac{\partial \vec{H}}{\partial t} = \nabla \times (\vec{u} \times \vec{H}) + \eta^2 \vec{H} - \frac{1}{Ne} \nabla \times [(\nabla \times \vec{H}) \times \vec{H}], \quad (6)$$

$$\frac{\partial}{\partial t} + (u \cdot \nabla)\, \rho = 0 , \quad (7)$$

$$\nabla \cdot \vec{u} = 0 , \quad \nabla \cdot \vec{H} = 0 , \quad (8)$$

In equation (6) e is the electron charge, N is the number density of the particles of the medium and $\eta = \frac{1}{\sigma \mu_e}$ is finite constant magnetic resistivity, σ being the electrical conductivity. Let $\delta\rho$, δp and $\vec{h} = (h_x, h_y, h_z)$ denote the perturbation respectively in density ρ, pressure p and the magnetic field $\vec{H}$ due to small disturbance given to the system which produce the velocity fields $\vec{u} = (u,v,w)$ in the fluid. Retaining only the linear terms in the perturbed quantities, we obtain the linearized perturbation equations.

$$(1+\lambda \frac{\partial}{\partial t})\, \rho \frac{\partial \vec{u}}{\partial t} = (1+\lambda \frac{\partial}{\partial t})\, [-\nabla \delta p + \vec{g}\,\delta\rho + \mu_e (\nabla \times \vec{h}) \times \vec{H}]$$
$$+ (1+\lambda_o \frac{\partial}{\partial t})\, [\mu \nabla^2 \vec{u} + (\nabla \vec{u}) \cdot \nabla\mu + (\nabla\mu) \cdot \nabla \vec{u}] , \quad (9)$$

$$\frac{\partial \vec{h}}{\partial t} = \nabla \times (\vec{u} \times \vec{H}) + \eta \nabla^2 \vec{h} - \frac{1}{Ne} \nabla \times [(\nabla \times \vec{h}) \times \vec{H}], \quad (10)$$

$$\frac{\partial}{\partial t} \delta\rho + (\vec{u} \cdot \nabla)\, \rho = 0 , \quad (11)$$

$$\nabla \cdot \vec{u} = 0 \quad \text{and} \quad \nabla \cdot \vec{h} = 0 \quad (12)$$

Analyzing the disturbance into normal modes, we assume that the perturbed quantities have a dependence on the space coordinates (x,y,z) and time t of the form

$$F(z) exp(ik_x x + ik_y y + nt) , \quad (13)$$

where $F(z)$ is some function of z, k_x and k_y $(k^2 = k_x^2 + k_y^2)$ and wave number in x- and y-directions and n denotes the rate at which a system departs from equilibrium.

It is assumed that $\vec{H} = (H_x, H_y, 0)$ is uniform. Making use of expression (13) in equations (9) - (12) and eliminating some of the variables, we finally obtain four equations for the four variables w, h_z, ζ and ξ:

$$n[\rho k^2 w - D(\rho D w)] - \frac{g k^2}{n} (D\rho) w$$
$$+ (ik_x H_x + ik_y H_y)\, (D^2 - k^2) h_z$$
$$+ \frac{1 + \lambda_o n}{1 + \lambda n} [\mu (D^2 - k^2)^2 w + 2D\, \mu (D^2 - k^2) D w$$
$$+ (D^2 + k^2)\, (D^2 \mu) w] = 0 , \quad (14)$$

$$[n\rho - \frac{1 + \lambda_o n}{1 + \lambda n} \mu (D^2 - k^2) - D\, \mu D] \zeta = (ik_x H_x + ik_y H_y) \xi \quad (15)$$

$$[n - \eta\, (D^2 - k^2)] \xi - (ik_x H_x + ik_y H_y) \frac{1}{Ne} (D^2 - k^2) h_z$$
$$= (ik_x H_x + ik_y H_y) \zeta , \quad (16)$$

$$[n - \eta\, (D^2 - k^2)] h_z + \frac{1}{Ne} (ik_x H_x + ik_y H_y) \xi$$
$$= (ik_x H_x + ik_y H_y) w , \quad (17)$$

where $D = \frac{d}{dz}$ and

$$\zeta = ik_x v - ik_y u \quad \text{and} \quad \xi = ik_x h_y - ik_y h_x , \quad (18)$$

are the z-components of the vector $\nabla \times \vec{u}$ and $\nabla \times \vec{h}$, respectively.

Boundary Conditions

In the present paper it is assumed that the fluid is bounded between two rigid planes at $z = 0$ and $z = d$, which are both assumed to be ideal conductors. Evidently, the fluid cannot have a normal velocity on the boundaries, and because of viscosity it cannot slip relative to the boundaries. It, therefore, follows that

$$w = 0, \quad \zeta = 0. \quad (19)$$

At the rigid boundaries, since the bounding surface is assumed to be ideally conducting, no disturbances within it can change the electromagnetic quantities outside. The normal component of the disturbance in the magnetic field must vanish at a rigid boundary. We must, therefore, have

$$h_z = 0 \text{ at } z = 0 \text{ and } z = d. \tag{20}$$

Finally, equation (17) in conjuction with equation (19) and (20), yields

$$\xi = 0 \text{ at } z = 0 \text{ and } z = d. \tag{21}$$

Variational Principle

Dropping the suffix z on h, we suppose that the solutions belonging to the characteristic value n_i are w_i, h_i, ζ_i, ξ_i, and the solution belonging to the characteristic value n_j are w_j, h_j, ζ_j, ξ_j. Multiplying equation (14) for i by w_j and integrating over the thickness of the fluid layer, we get

$$n_i \int_o^d [\rho k^2 w_i - D(\rho D w_i)] w_j \, dz - \frac{gk^2}{n} \int_o^d (D\rho) w_i w_j \, dz$$
$$+ (ik_x H_x + ik_y H_y) \int_o^d (D^2 - k^2) h_i w_j \, dz$$
$$+ \frac{1 + \lambda_o n}{1 + \lambda n} \int_o^d [\mu (D^2 - k^2)^2 w_i + 2(D\mu)(D^2 - k^2) D w_i$$
$$+ (D^2 + k^2)(D^2 \mu) w_i] w_j \, dz = 0 \ , \tag{22}$$

Integrating by parts once or repeatedly and using the boundary conditions (19)-(21), we obtain after setting $i = j$

$$n[I_1 - I_3 + I_4 - I_5] - \frac{gk^2}{n} I_2$$
$$+ \frac{1 + \lambda_o n}{1 + \lambda n} [I_6 - I_7] + \eta (I_8 - I_9) = 0 \ , \tag{23}$$

where

$$I_1 = \int_o^d \rho [(Dw)^2 + k^2 w^2] \, dz \ , \tag{24}$$

$$I_2 = \int_o^d (D\rho) w^2 \, dz \ , \tag{25}$$

$$I_3 = \int_o^d [(Dh)^2 + k^2 h^2] \, dz \ , \tag{26}$$

$$I_4 = \int_o^d \xi^2 \, dz \ , \tag{27}$$

$$I_5 = \int_0^d \rho \zeta^2 \, dz \ , \tag{28}$$

$$I_6 = \int_o^d \mu [\{(D^2 + k^2) w\}^2 + 4k^2 (Dw)^2] \, dz \ , \tag{29}$$

$$I_7 = \int_o^d \mu [(D\zeta)^2 + k^2 \xi^2] \, dz \ , \tag{30}$$

$$I_8 = \int_o^d [(D\xi)^2 + k^2 \xi^2] \, dz \ , \tag{31}$$

$$I_9 = \int_o^d [(D^2 h)^2 + 2k^2 (Dh)^2 + k^4 h^2] \, dz \ , \tag{32}$$

Consider a variation δn in n consequent to the variations δw, δh, $\delta \zeta$, $\delta \xi$ related to it through perturbed form of equation (15)-(17) in w, h, ζ, ξ respectively compatible with the boundary conditions. By proceeding along the usual lines we can show that, to first, order, $\delta n = 0$.

Fluid Layer of Varying Density

Suppose that the density in the Oldroydian fluid layer is continuously stratified exponentially along the vertical i.e.

$$(z) = \rho_1 \exp(\beta z) \ , \tag{33}$$

where ρ_1 and β are constants, ρ_1 being the density at the lower boundary.

We assume that the coefficient of kinematic viscosity v_o is constant, so that

$$u(z) = v_o \rho_1 \exp(\beta z) \tag{34}$$

For mathematical simplicity, it is also assumed that

$$|\beta d| \ll 1 \ , \tag{35}$$

which implies that the difference between the densities at two neighbouring points is much less than the average density of the fluid (19)-(21), consistent with the conditions to be satisfied at the boundaries (19)-(21), the trial solutions for $w(z)$, $h(z)$, $\zeta(z)$ and $\xi(z)$ are taken as

$$\begin{bmatrix} w(z) = A_1 \sin lz & \zeta(z) = A_3 \sin lz \\ h(z) = A_2 \sin lz & \xi(z) = A_4 \sin lz \end{bmatrix} \quad (36)$$

where A_1, A_2, A_3 and A_4 are constant and $1 = \frac{m\pi}{d}$, m being an integer.

Substituting these trial solutions in equation (22), using Equations (33)-(36) and eliminating the constants A_1, A_2, A_3 and A_4 by using equations (15)-(17), we finally obtain the dispersion relation

$$n^2 - \frac{g\beta k^2}{l^2 + k^2} + \frac{1 + \lambda_o n}{1 + \lambda n} \cdot n\nu_o(l^2 + k^2)$$
$$+ n(\vec{k} \cdot \vec{V})^2 \left[\{n + \eta(1^2 + k^2)\}\right.$$
$$\left[\frac{(\frac{\vec{k}\cdot\vec{H}}{Ne})^2(l^2+k^2)\left\{n+\nu_o(l^2+k^2)\frac{1+\lambda_o n}{1+\lambda n}\right\}}{(\vec{k}\cdot\vec{V})^2 + \{n+\eta(l^2+k^2)\}\left\{n+\nu_o(l^2+k^2)\frac{1+\lambda_o n}{1+\lambda n}\right\}}\right]^{-1} = 0 , \quad (37)$$

where

$$\vec{k} \cdot \vec{V} = \frac{k_x H_x + k_y H_y}{\sqrt{\rho_1}} \text{ and } \frac{\vec{k}\cdot\vec{H}}{Ne} = \frac{k_x H_x + k_y H_y}{Ne} \quad (38)$$

On writing

$$Y = \frac{n}{\sqrt{g\beta l}},\ M = \frac{\nu_o l}{\sqrt{g\beta}},\ T_o = \lambda_o l\sqrt{g\beta},\ T = \lambda_l\sqrt{g\beta}$$
$$x = \frac{k}{l},\ V_l = \frac{H_x}{\sqrt{g\beta\rho_1}},\ V_2 = \frac{H_y}{\sqrt{g\beta\rho_1}},$$
$$L_1 = \frac{H_x l}{Ne\sqrt{g\beta}},\ L_2 = \frac{H_y l}{Ne\sqrt{g\beta}} \text{ and } N = \frac{\eta l}{\sqrt{g\beta}} \quad (39)$$

$V = (V_1 \cos\theta + V_2 \sin\theta)^2$,
$L = (L_1 \cos\theta + L_2 \sin\theta)^2$

we obtain the non-dimensional form of the dispersion relation as

$$T^2Y^7 + 2Y^6T[1 + (1 + x^2)(MT_o + NT)]$$
$$+ Y^5\Big[1 + 2T^2Vx^2 + (1 + x^2)$$
$$\{TLx^2 + 2MT + N + MT_o(2 + MT_o)\}$$
$$+ (1 + x^2)^2\{TN^2 + MNT_0(1 + 4T)\} - \frac{Tx^2}{1 + x^2}\Big]$$
$$+ Y^4\Big[Tx^2(4VT - MT_o - 2NT) + (1 + x^2)$$
$$\{2M + N + MN(3T_o + 4T) + 2Tx^2(MT_o(V+L) + NV + L)\}$$
$$+ 2(1 + x^2)^2\{MT_o(M + N(TN + MT_o)) + TN^2\} - \frac{2Tx^2}{1+x^2}\Big]$$
$$+ Y^3\Big[x^2(2V + T^2V^2 - M(T + T_o) + 4TN)$$
$$+ N(1+x^2)(N + T_o + 2T) + x^2(1+x^2)(N^2 + NV(1 + 4T)$$
$$- TN(TN + 2MT_o) - Tx^2(V - L) + L^2) + MN(1 + x^2)^2$$
$$\{5 + 2T_o(1+VTx^2) + (1+x^2)(3MT_o + 2N(T+T_o) + NM^2$$
$$+ T_oMN(1+x^2)^2\} + MLx^2(1 + x^2)^2$$
$$\{MT_o^2(1 + x^2) + 2(T + T_o)\} - \frac{x^2}{1 + x^2}\Big]$$
$$+ Y^2\Big[2x^2(VT - N) - x^4\{TL + NV(2 + T^2) + x^2(1 + x^2)$$
$$\{MN(2T + T_o) + M(2V + TT_oLx^2 - 2N^2\} + Mx^2(1 + x^2)^2$$
$$\{2NV(T + T_o) + 2L(1 + MT_o) - TT_oN^2\}$$
$$+ 2MN(1 + x^2)^3\{N + M(N(1 + x^2) + 1)\}\Big]$$
$$+ Y\Big[M^2(1 + x^2)^3\{N^2 + x^2(N^2 + L)\} + x^2(1 + x^2)$$
$$\{M\{2N + Lx^2 + N(1 + x^2)(2V + N(T + T_o))\} - V^2\}$$
$$- x^4(L + 2TNV + \frac{V^2}{1 + x^2})\Big]$$
$$- x^2\Big[MN(1 + x^2)^2 + x^2(NV + ML)\Big] = 0 \quad (40)$$

where θ is the angle between the wave vector $\vec{k}$ and $\vec{H}$, and V_1 and V_2 are Alfven velocities.

Conclusions

The dispersion relation (40) is quite complex and a direct solution is obviously quite difficult to obtain. As we are interested in knowing the growth rate of the unstable modes, we have performed numerical calculations of equation (40) to locate the roots of Y against x for several values of the parameters M, L_1, L_2, V_1, V_2, T_o, T, N and the angle θ. These calculations

Table 1

Values of growth rate (positive real value of Y) against wave numer x, $T_o = 0.1$, 0.3, 0.5, (here $V_1 = V_2 = 0.25$, $L_1 = L_2 = 1.0$, $N = 1.0$, $T = 0.6$, $M = 0.1$ and $\theta = 45°$)

x	Values of growth rate		
	$T_0 = 0.1$	$T_0 = 0.3$	$T_0 = 0.5$
0.0	0.0000	0.0000	0.0000
0.2	0.3465	0.3454	0.3441
0.4	1.0139	1.0124	1.0107
0.6	1.5890	1.5915	1.5936
0.8	1.9442	1.9483	1.9514
1.0	2.0910	2.0923	2.0918
1.2	2.0859	2.0823	2.0763
1.4	1.9878	1.9796	1.9689
1.6	1.8418	1.8306	1.8171
1.8	1.6781	1.6655	1.6510
2.0	1.5144	1.5017	1.4876

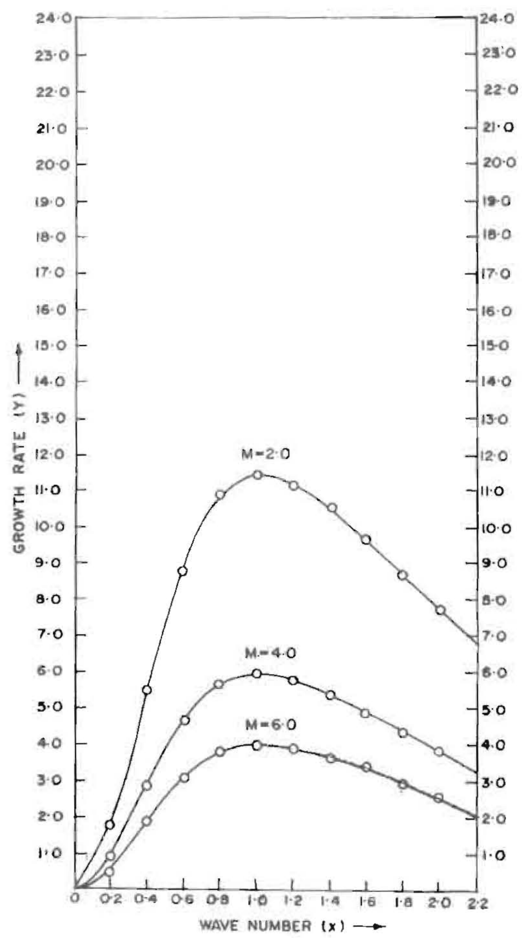

Figure 1.

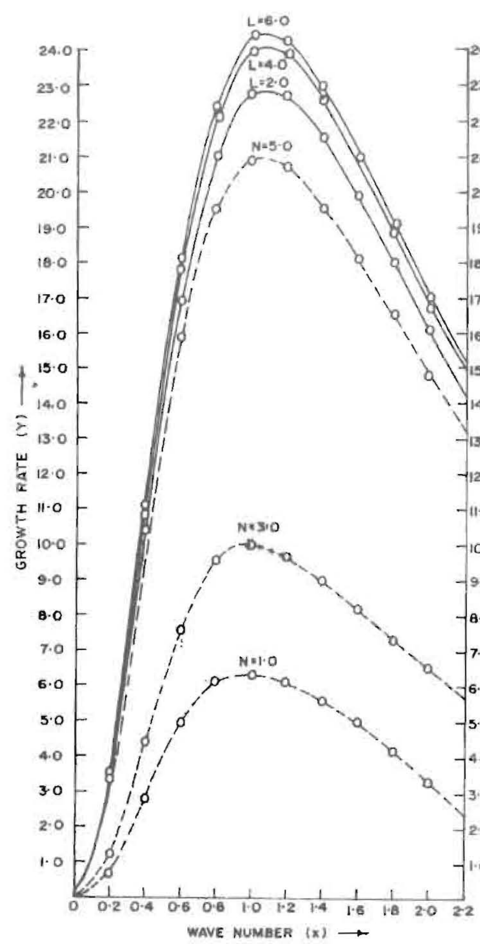

Figure 2.

different values of M, T_0, L_1, L_2, V_1, V_2, N and θ. From Table 1 it is seen that the growth rate Y decreases for the same x (wave number) on increasing the parameter T_0 characterizing the elasticity on the fluid when the other parameters are kept fixed. The effects of elasticity is, therefore, stabilizing. Also by elasticity for larges values of wave number x since the growth rate is seen to decrease with increasing x. From Figure 1 we see that as M (viscosity) increases, Y (the growth rate) decreases for the same x (wave number) indicating that the influence of viscosity is stabilizing. It can also be seen from Figure 1 that viscosity can also stabilize the system completely for large wave numbers for the growth rate is seen to decrease for large x. Further it is seen that the more the fluid is viscous the smaller is the wave number for which the system can be completely stabilized. From Figure 2 it is seen that Y (growth rate) increases as L (Hall current) increases for the same x, thus indicating the destabilizing character of the Hall current. Figure 2 shows that Y increases as N (finite conductivity increases for the same x. The influence of finite conductivity is thus also destabilizing.

We may thus conclude that the viscosity has a stabilizing effects while Hall currents and finite conductivity have a destabilizing influence on the stability of a viscoelastic fluid. In that respect the results are the same as obtained earlier for Newtonian fluid.

Acknowledgements

Authors are gratful to the referees for some useful suggestions which have helped in inproving the paper.

References

1. Rayleigh Lord.: Proc. Lond. Maths. Soc., Vol. 14,(1983), 170.
2. Taylor G.I.: Proc. Roy. Soc. Lond., Vol. A 201, (1950), 192.
3. Chandrasekhar S.:Hydrodynamic and Hydromagnetic Stability, Clarendon Press, Oxford, 1961.
4. Kruskal M. and Schawarzchild M.: Proc. Roy. Soc. Lond., Vol. A 223, (1954), 348.
5. Hide R.: Proc. Roy. Soc., Vol. 223, (1954), 376.
6. Hosking R.J.: J. Plasma Phy., Vol. 2, (1968), 613.
7. Singh S. and Tandon J.N.: J. Plasma Phy., Vol. 3, (1969), 633.
8. Ariel P.D.: J. Plasma Phy., Vol. 5, (1970), 523.
9. Bhowmik G.: J. Plasma Phys., Vol. 7, (1972), 117.
10. Bhatia P.K. and Steiner J.M.: Aust. J. Phys., Vol. 25, (1972), 695.
11. Vest C.M. and Arpaci V.S.: J. Fluid Mech., Vol. 36, (1969), 613.
12. Bhatia P.K.: J. Plasma Phy., Vol. 11, (1974), 1.
13. Sharma B.M.: Janabba, Vol. 14, (1984),33.
14. Samria N.K., Reddy M.U.S. and Prasad R.O.: Astrophys Space Sci., Vol. 172, (1990), 231.
15. Gupta A. and Bathia P.K.: Astrophys. Space Sci., Vol. 181, (1991), 103.
16. Ali A.: Ph.D. Thesis 'Some Hydromagnetic Stability Problems, J.N. Vyas University, Jodhpur, India, 1992.

Recibido el 11 de Febrero de 1993
En forma revisada el 11 de noviembre de 1994