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Dr. Humberto Fernández Morán
"Mis luchas son las del Quijote,
batallas que uno pierde, pero
gana la causa".

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Error-in-Variables for Special Models Describing Brittle-Ductile Transition
Error-in-Variables for Special Models

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Abstract

In this work it shows the application of the method of error-in-variable (EIV), based methodology to fit the envelope in the principal effective stresses plane considering the envelope describing brittle-ductile transition and pore collapse (cap model). The parametric equations of the failure envelope can be obtained introducing the concept of poroelasticity of Biot, representing the state of stress by the effective mean stress and the equivalent stress . A nonlinear algebraic form represents the envelope equations delimiting the brittle-to-ductile transitional region of the failure or yield envelope. To illustrate the application of this methodology, we use the mechanical data for the brittle strength and compactive yield stress for Bentheim sandstone and the normalized principal stresses for ten different sandstones describing brittle-ductile transition behavior. Results demonstrate that the EIV method provides a simple way to obtain the parametric representation of the envelope in the plane of principal effective stress, describing the deformation mechanism of the tested rock that involves the pore collapse effect.
Keywords: cap model; EIV; failure criterion; pore collapse; poroelasticity.

Error-en-las-Variables para Modelos Especiales Describiendo la Transición Frágil-Dúctil
Error-en-las-Variables para Modelos Especiales

Resumen

En este trabajo se muestra la aplicación de la metodología basada en el método de error en las variables (EIV) para ajustar la envolvente en el plano de los esfuerzos efectivos principales, considerando la envolvente y describiendo la transición frágil a dúctil y el colapso de poros (modelo cap). Las ecuaciones paramétricas de la envolvente de rotura se obtuvieron introduciendo el concepto de poro elasticidad de Biot, representando el estado de esfuerzos por el esfuerzo efectivo promedio y el esfuerzo equivalente. Una forma algebraica no lineal representa las ecuaciones de la envolvente, delimitando la región transicional de frágil a dúctil de la envolvente de rotura o cedencia. Para ilustrar la aplicación de esta metodología, se usó la información de la resistencia frágil y el esfuerzo de cedencia a la compactación para la arenisca Bentheim y los esfuerzos principales normalizados para diez diferentes areniscas en las

que se describe el comportamiento de transición frágil a dúctil. Los resultados demuestran que el método EIV proporciona una forma sencilla de obtener la representación paramétrica de la envolvente en el plano de tensión efectiva principal, describiendo el mecanismo de deformación de la roca ensayada que implica el efecto de colapso de los poros.
Palabras clave: colapso de poro; criterio de falla; EIV; modelo cap; poroelasticidad.

Erro variável para modelos especiais que descrevem a transição frágil-dúctil
Variável de erro para modelos especiais

Resumo

Este artigo mostra a aplicação da metodologia baseada no método error in variables (EIV) para ajustar o envelope no plano das principais tensões efetivas, considerando o envelope e descrevendo a transição frágil-dúctil e o colapso dos poros (modelo cap). As equações paramétricas do envelope de rutura foram obtidas através da introdução do conceito de elasticidade dos poros de Biot, representando o estado de tensões pela tensão efetiva média e pela tensão equivalente. Uma forma algébrica não linear representa as equações do envelope, delimitando a região de transição frágil a dúctil do envelope de quebra ou rendimento. Para ilustrar a aplicação desta metodologia, foram utilizadas as informações de resistência frágil e tensão de rendimento à compactação para o arenito de Bentheim e as tensões principais normalizadas para dez arenitos diferentes, nas quais o comportamento de transição frágil para dúctil é descrito. Os resultados mostram que o método EIV fornece uma maneira simples de obter a representação paramétrica do envelope no plano de tensão efetivo principal, descrevendo o mecanismo de deformação da rocha testada que envolve o efeito de colapso dos poros.
Palavras-chave: colapso dos poros; critério de reprovação; EIV; Modelo da PAC; poroelasticidade.

Introducción

A failure criterion is usually a relation between the principal effective stresses components and represents a limit for instability or failure. In the presence of pore fluids, the mechanical properties of porous rocks depend on both pore fluid pressure and confining pressure according to the Terzagui effective pressures law (Terzaghi, 1943), which can be used for many rock types and physical properties in its more general form as , where   is a constant rock type (usually referred to as Biot's constant (Biot, 1941) that takes on different values for different materials but is always less than or equal to one (Smits et al., 1988), is the effective pressure, is the confined pressure and is the porous pressure. Rocks fail under stress in different modes depending on the nature and magnitude of the applied stress and on the properties of the rock itself. Higher-porosity sedimentary rocks are found to compact and lose porosity under applied stress. When fluids are produced, the effective stress increases in the reservoir and may exceed the compressive strength of the rock matrix. The rock matrix collapses into the pore space in a mechanism known as "pore collapse" (Nur and Byerlee, 1971; Addis, 1987). Initially, lower effective stresses cause little or no pore collapse, and deformation is recoverable (elastic). But as effective stress increases, the reservoir undergoes unrecoverable pore collapse, accompanied by elastic deformation, which reduces porosity and permeability of the reservoir. The behavior of some porous rocks undergoes compaction and elastoplastic constitutive relations can model pore collapse. But the behavior afterward is not perfectly plastic, and a hardening rule is needed for a completed description. The behavior of porous sandstones undergoing pore collapse and compaction has been found to fit an approximately elliptical "cap model" (Chen and Mizumo, 1990).

A cap model consists of a failure surface for a perfectly plastic material response and an elliptic strain-hardening cap that extends isotropically about the hydrostatic axis (Wong et al., 1997) parametric representation describing the deformation mechanism of the porous rock can be achieving by a method of curve fitting. The objective of this study is to create the envelope of a cap-type behavior high porosity sedimentary rock in such a way that the derived values of the effective mean stress   and the equivalent stress   are considered having experimental error. If the effect of the principal intermediate stress is considered having not to influence the rock strength (triaxial test) this failure criterion can be expressed in terms of the effective mean stress and the equivalent stress, which can be represented by (Zambrano-Mendoza, 2004).

$$h(\sigma_m, \sigma_{eq}) = 0 \tag{1}$$

Since this work is an extension of the use of the statistical method of error-in-variables to fit the failure envelope in the principal stress plane using experimental data from a triaxial test (Zambrano-Mendoza, 2004; Zambrano-Mendoza et al., 2023), it was utilized the established formulations to developed an appropriate methodology to describe the deformation mechanism of those sedimentary reservoir rocks, which involve brittle-transition-ductile behavior in they deformation mechanism.

In a previous earlier review of the use of the statistical method of error-in-variable found the improvement achieve in the statistical method of EIV since Deming (1943), first formulation. Previously it was reviewing the least square method and others some variants of the least square method, such as the orthogonal regression method (Keles and Altun, 2016; Recio-Lopez, 2021). Even though that improvement of the least square method there is still a dependency of one of the variables. Here it is summarized the different improvements in the error-in-variable statistical method used in this work (York, 1966; Willianson, 1968; O'Neil et al., 1969; Southwell, 1969; Britt and Luecke, 1973; Peneloux et al., 1976; Reilly and Patino-Leal, 1981; Schwetlick and Tiller, 1985; Valkó and Vajda, 1987; Liebman and Edgar, 1988; Edgar et al., 1990; Esposito and Floudas, 1998; Anand and Kumar, 2015; Kumar and Kumar, 2011) as well as described the EIV method of curve fitting and developed a variant suitable for fitting failure envelopes in the Mohr plane (Zambrano-Mendoza et al., 2003) and in the principal stress plane (Zambrano-Mendoza, 2004; Zambrano-Mendoza et al., 2023) applying it to a well-established set of data.

In this work, it is proposing to use the statistical method of error-in-variables (EIV) to obtain the parametric representation of the failure envelope in the principal effective stress plane providing an accurate way of obtaining the parametric representation of the failure envelope describing the deformation mechanism. Next; it is describing the application of the EIV method of curve fitting in the principal effective stress plane to a previously established set of data (Klein et al., 2001, Zhang et al., 2000).

Materials and Methods

Application of EIV method to fit the failure envelope in the principal effective stress plane

The group of parametric equations of the failure envelope developed in the principal stress plane can be modified in a very simple way by introducing the concept of poroelasticity of Biot (Biot, 1941). The state of stress can be represented by the equivalent stress and the effective mean stress as stress invariants that are independent of coordinate rotation. These two invariants can be written as (Biot, 1941):

$$\sigma_m = (\sigma_1 + 2\sigma_3)/3 - \alpha p_p \tag{2}$$

And from von Mises (1913):

$$\sigma_{eq} = \sigma_1 - \sigma_3 = \sqrt{3J_2} \tag{3}$$

Where (p_p) is reservoir pore pressure, (σ_1) the major principal stress and (σ_3) the minor principal stress and $\alpha = 1 - {c_{ma}}/{c_b}$ (c_{ma}) is the matrix compressibility; (c_b) is the bulk compressibility. For porous sandstones: $c_{ma} \leq c_b$, so $\alpha = 1$.

Equations 2 and 3 represent the mean effective stress and the equivalent stress respectively, and (J_2) in MPa^2, is the second invariant of the stress deviator tensor. Using those equations and the EIV approach, it was generating an algebraic solution for the material that will satisfy the equation of the envelope given in implicit form as (Zambrano-Mendoza, 2004):

$$h\left(\hat{\sigma}_m, \hat{\sigma}_e, \underline{\theta''}\right) = 0 \tag{4}$$

Where $\left(\hat{\sigma}_e\right)$ is the reconciled equivalent stress and $\left(\hat{\sigma}_m\right)$ reconciled effective mean stress and $\left(\underline{\theta''}\right)$ is the unknown parameter.

A similar procedure of the EIV approach used in $\left(\sigma_3, \sigma_1\right)$ stress plane could be applied. The goal is to find the optimum parameters, at which the sum of necessary corrections squared:

$$J = \sum d_{pi}^2 \tag{5}$$

(J) is minimum, expressing the squared distance $\left(d_{pi}^2\right)$ in terms of the mean effective stress and the equivalent stress as follow:

$$d_{p_i}^2 = \left(\hat{\sigma}_{mi} - \sigma_{mi}\right)^2 + \left(\hat{\sigma}_{eqi} - \sigma_{eqi}\right)^2 \tag{6}$$

With $\left(\hat{\sigma}_{ei}\right)$ is the ith measurement of the reconciled equivalent stress and $\left(\hat{\sigma}_{mi}\right)$ the ith measurement of the reconciled effective mean stress, while $\left(\sigma_{ei}\right)$ is the ith measurement of the equivalent stress and $\left(\sigma_{mi}\right)$ the ith measurement of the effective mean stress.

Because the failure envelope is fitting in the principal effective stress plane the EIV algorithm used for the principal's stress plane (Zambrano-Mendoza *et al*., 2023), which can be modified as follows:

EIV algorithm

Thus, vector **a**, can be expressed as:

$$\mathbf{a} = -\frac{\partial h\left(\hat{\sigma}_{mi}, \hat{\sigma}_{eqi}, \underline{\theta''}\right)}{\partial \hat{\sigma}_{eq}}\mathbf{i} + \frac{\partial h\left(\hat{\sigma}_{mi}, \hat{\sigma}_{eqi}, \underline{\theta''}\right)}{\partial \hat{\sigma}_{m}}\mathbf{j} \tag{7}$$

While vector **b** is expressed as:

$$\mathbf{b} = \left(\hat{\sigma}_{mi} - \sigma_{mi}\right)\mathbf{i} + \left(\hat{\sigma}_{eqi} - \sigma_{eqi}\right)\mathbf{j} \tag{8}$$

The orthogonality between vector **a** and **b** is expressed as:

$$-\left(\hat{\sigma}_{mi} - \sigma_{mi}\right)\frac{\partial h\left(\hat{\sigma}_{mi}, \hat{\sigma}_{eqi}, \underline{\theta''}\right)}{\partial \hat{\sigma}_{eq}} + \left(\hat{\sigma}_{eqi} - \sigma_{eqi}\right)_i \frac{\partial h\left(\hat{\sigma}_{mi}, \hat{\sigma}_{eqi}, \underline{\theta''}\right)}{\partial \hat{\sigma}_{m}} = 0 \tag{9}$$

The system of Equations 4 and 9 can be solved simultaneously using either excel VBA or Mathematics Software, to obtain the reconciled stress $\left(\hat{\sigma}_m, \hat{\sigma}_{eq}\right)$ at any parameter vector $\underline{\theta''}$ which have been already defined, evaluating the objective function. A nonlinear function is considered to represent the failure envelope at the principal effective stress plane.

Applying the method to special model suitable for describing brittle-ductile transition in yield envelope

Using the concept of effective stresses state in the plane or invariants, can be obtain the failure or yield envelope considering pore collapse effect. From the EIV approach it can be generating an algebraic solution for the brittle-ductile transition behavior of the material that will satisfy Equation 4. The parametric representation proposed to fit the failure envelope in the principal effective stress plane is given by a four parameters equation fitting the envelope in such a way that describe the deformation mechanism drawing a curve, which connect the brittle region to the ductile region defining the cap model. Then, Equation 4 can be written as:

$$h\left(\hat{\sigma}_{mi}, \hat{\sigma}_{eqi}, \underline{\theta''}\right) = \theta''_0 + \theta''_1\ \hat{\sigma}_{mi} + \theta''_2\ \hat{\sigma}^2_{mi} - \theta''_3\ \hat{\sigma}_{eqi} - \hat{\sigma}^2_{eqi} = 0 \tag{10}$$

$$\left(\theta''_1 + 2\theta''_2\hat{\sigma}_{mi}\right)\left(\hat{\sigma}_{eqi} - \sigma_{eqi}\right) - \left(\theta''_3 + 2\hat{\sigma}_{eqi}\right)\left(\sigma_{mi} - \hat{\sigma}_{mi}\right) = 0 \tag{11}$$

The θ''_0 , θ''_1 , θ''_2 and θ''_3 are the unknown coefficients, while the rest of parameters have been already defined previously. Solving the system of Equations 10 and 11 it can be obtaining the squared distance by Equation 6. Due to the cumbersome nature of $\hat{\sigma}_{3i}$ roots the squared distance, d^2_{pi} is not showed for the equations.

Results and Discussions

To illustrate the applicability of the EIV method to fit the failure envelope for special model describing brittle-ductile transition with pore collapse effect in the principal effective stress plane, here it is used the previously set experiments (Klein *et al*., 2001; Zhang *et al*., 2000). A nonlinear parametric function was considered to fit the envelope. The goal was to obtain the failure envelopes for those rocks in the principal effective stress plane, using all the available information. Table 1 contains the optimal parameters for normalized data for 10 sandstones. The parameters describe the failure envelope located nearest to the respective pairs of the mean-effective and equivalent-stress measurements obtained from the data measured under various confining pressures for the critical pressure normalized data for 10 different sandstones, while Table 2 contains the optimal parameters for Bentheim sandstone. The parameters describing the yield envelope located nearest to the respective pairs of the mean-effective and equivalent-stress measurements obtained from the measured data, including critical stress at the onset of dilatancy and compactive yield stress at the onset of shear-enhanced compaction (11 test samples) and considering only compactive yield stress defining a cap model (7 test samples).

Table 1. Optimal parameters determined from the EIV method considering pore collapse effect for a normalized data.

Nonlinear 4 parameters model				
$f\left(\hat{\sigma}'_m, \hat{\sigma}_{eq}, \underline{\theta''}\right) = \theta''_0 + \theta''_1\hat{\sigma}'_m + \theta''_1\hat{\sigma}'^2_m - \theta''_3\hat{\sigma}_{eq} - \hat{\sigma}_{eq}{}^2$ |				
	θ''_0	θ''_1	θ''_2	θ''_3
Test rock	(-)	(-)	(-)	(-)
Normalized data from 10 sandstones	5.1E-13	20.7	-18.2	10.4

35 test samples, $\left(\hat{\sigma}'_m\right)$ = reconciled measured mean effective stress, $\left(\hat{\sigma}_{eq}\right)$ = reconciled measured equivalent stress, θ''_0, θ''_1, θ''_2, θ''_3= unknown coefficients.

Table 2. Optimal parameters determined from the EIV method for bentheim sandstone considering pore collapse effect.

Nonlinear 4 parameters model				
$f\left(\hat{\sigma}'_m, \hat{\sigma}_{eq}, \underline{\theta''}\right) = \theta''_0 + \theta''_1\hat{\sigma}'_m + \theta''_1\hat{\sigma}'^2_m - \theta''_3\hat{\sigma}_{eq} - \hat{\sigma}_{eq}{}^2 \mid$				
	θ''_0	θ''_1	θ''_2	θ''_3
Test rock	MPa^2	MPa	(-)	MPa
Bentheim sandstone[a]	7.9E-13	1717.4	-3.7	652
Bentheim sandstone[b]	1.4E+5	629.6	-1.9	562.4

a: 11 test samples, b: 7 test samples, $\left(\hat{\sigma}'_m\right)$ = reconciled measured mean effective stress, $\left(\hat{\sigma}_{eq}\right)$ = reconciled measured equivalent stress, θ''_0, θ''_1, θ''_2, θ''_3= unknown coefficients.

The possible use of these parameters is within a computational algorithm, where a stress state needs to be tested for failure. The listed parameter, can minimize the likelihood of making a wrong judgment (i.e. declaring a failure state as safe or a safe state as failure). The standard deviation is the sum of the squared distances between the failure envelope and each pair of measurements (objective function values); the standard deviation is given by:

$$S.D = \sqrt{\frac{J}{n-4}} \tag{12}$$

The standard deviations result to be 0.06 for the normalized data for 10 different sandstones, while the standard deviation for Bentheim sandstone is 12.2 MPa for whole envelope and 10.2 MPa for the case when considering only compactive yield stress to define a cap model.

Figure 1 shows the parametric representation of the envelope of the normalized principal stresses describing brittle/ductile transition behavior for 10 different sandstones (Zhang *et al*., 2000). The solid black line represents the optimal parametric representation of the yield envelope.

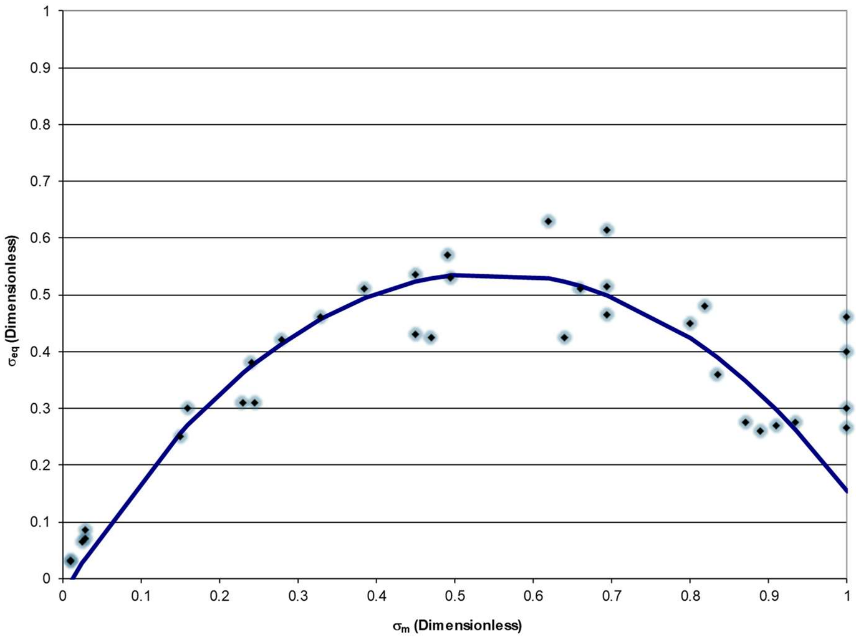

Figure 1. Parametric representation of the failure envelope in the principal stress plane considering pore collapse effect using EIV method for normalized data for 10 sandstones (nonlinear four-parameters model). Where: effective mean stress (σ_m) and the equivalent stress (σ_{eq}).

Figure 2 shows the parametric representation of the envelope including critical stress at the onset of dilatancy and compactive yield stress at the onset of shear-enhanced compaction (11 test samples) and considering only compactive yield stress defining a cap model (7 test samples) for Bentheim sandstone (Anand and Kumar, 2015). It should be emphasizing that according with some previous work a 400 MPa it is the critical pressure.

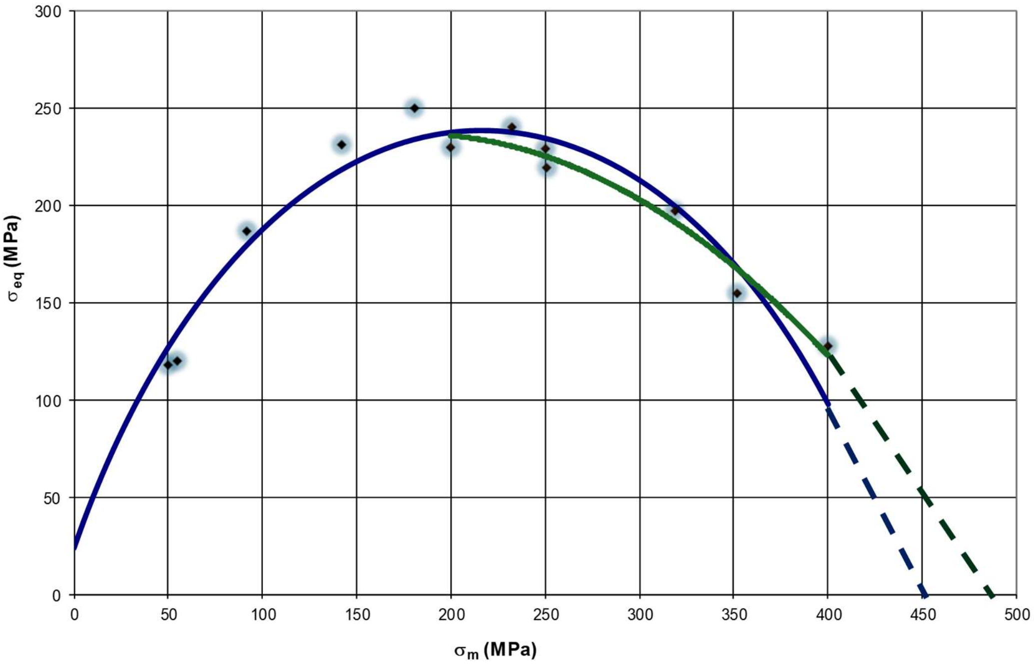

Figure 2. Parametric representation of the failure envelope in the principal stress plane considering pore collapse effect using EIV method for Bentheim sandstone (nonlinear four-parameters model) . Based on conventional triaxial test data. (11 test samples) dark curve and (7 test samples) ligth curve. Where: effective mean stress (σ_m) and the equivalent stress (σ_{eq}).

Conclusions

The EIV method can provide a simple way to obtain the parametric representation of the envelope in the principal effective stress plane describing the deformation mechanism of the tested rock involving pore collapse effect. The parameters describing the yield envelope located nearest to the respective pairs of the mean-effective and equivalent-stress measurements.

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