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R. K. Sharma

Department of Mathematics
Adeyemi College of Education
University of Ife
Ondo
Nigeria.

Another Watson's Theorem For Double Series

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RESUMEN

El objeto de este trabajo es demostrar la siguiente fórmula de suma:

$$F\left[\begin{matrix} p; & 2\lambda_1+1, \lambda_1; 2\lambda_2+1, \lambda_2; 1, 1 \\ \frac{1}{2}(p+2\lambda_1+2\lambda_2+3); & 2\lambda_1; 2\lambda_2; \end{matrix}\right]$$

$$= \frac{\Gamma(\frac{1}{2})\,\Gamma(-\frac{1}{2}-\frac{1}{2}p)\,\Gamma(\frac{3}{2}+\lambda_1+\lambda_2+\frac{1}{2}p)}{\Gamma(\frac{1}{2})\,\Gamma(\frac{1}{2}+\frac{1}{2}p)\,\Gamma(\frac{3}{2}+\lambda_1+\lambda_2-\frac{1}{2}p)}$$

$R(p)<1$, $R(\lambda_1)>0$ y $R(\lambda_2)>0$

SUMMARY

The object of the present paper is to prove the following sumation formula:

$$F\left[\begin{matrix} p; 2\lambda_1+1, \lambda_1; 2\lambda+1, x\lambda_2; 1, 1 \\ \frac{1}{2}(p+2\lambda_1+2\lambda_2+3); 2\lambda_1; 2\lambda_2; \end{matrix}\right]$$

$$= \frac{\Gamma(\frac{1}{2})\,\Gamma(-\frac{1}{2}-\frac{1}{2}p)\,\Gamma(\frac{3}{2}+\lambda_1+\lambda_2+\frac{1}{2}p)}{\Gamma(\frac{1}{2})\,\Gamma(\frac{1}{2}+\frac{1}{2}p)\,\Gamma(\frac{3}{2}+\lambda_1+\lambda_2-\frac{1}{2}p)},$$

$R(p)<1$, $R(\lambda_1)>0$ $R(\lambda_2)>0$.

1. INTRODUCTION:- Recently Sharma [1] has proved the summation theorem

$$F\left[\begin{matrix} \alpha, p; y_1; y_2; 1, 1 \\ 2\alpha, \frac{1}{2}(p+y_1+y_2+1); -; -; \end{matrix}\right]$$

$$= \frac{\Gamma(\frac{1}{2})\,\Gamma(\frac{1}{2}+\alpha)\,\Gamma(\frac{1}{2}+\frac{1}{2}p+\frac{1}{2}y_2)}{\Gamma(\frac{1}{2}+\frac{1}{2}p)\,\Gamma(\frac{1}{2}+\frac{1}{2}y_1+\frac{1}{2}y_2)}$$

$$\frac{\Gamma(\frac{1}{2}+\alpha\,\frac{1}{2}p-\frac{1}{2}y_1-\frac{1}{2}y_2)}{\Gamma(\frac{1}{2}-\frac{1}{2}p+\alpha)\,\Gamma(\frac{1}{2}-\frac{1}{2}y_1-\frac{1}{2}y_2+\alpha)} \quad (1)$$

valid for $R(2\alpha - p - y_1 - y_2 + 1) > 0$

(1) is a Watson theorem for hypergeometric series of two variables. In case $y_1 = 0$ or $y_2 = 0$ in (1), it reduces to Professor Watson's theorem (see [4; p.54 (2.3.3.13)]).

The object of this paper is to prove another Watson's theorem for hypergeometric series of two variables. Professor Carlitz [3] has proved a Saalschützian theorem for double series. The following notation due to Burchnall and Chaundy [2] has been used to represent the hypergeometric series of higher order and of two variables.

(2)

$$F\left[\begin{matrix}(a_p);\ (b_q);\ (c_r);\ x\ ,\ y\\ (d_s);\ (e_n);\ (f_k);\end{matrix}\right]$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{[(a_p)]_{m+n}[(b_q)]_m[(c_r)]_n\ x^m y^n}{[(d_s)]_{m+n}[(e_h)]_n[(f_k)]_n\ m!\ n!} \qquad (2)$$

where (a) and $(a\ )\ m\text{-}n$ Will mean the sequence

$a_1,\ldots\ldots a_p$ and the product $(a_1)_{m+n}\ldots\ldots\ldots(a_p)_{m+n}$ respectively.

In the investigation we use the formulae due to Gauss (see[4, p.28 (1.7.6.)])

$${}_2F_1\left[\frac{1}{2}n,\frac{1}{2}-\frac{1}{2}n;\ \alpha+\frac{1}{2};1\right]=\frac{2^n(\alpha)_n}{(2\alpha)_n}, \qquad (3)$$

valid for $R(\alpha)>0$ and n is a positive integer, Sharma[1, p. 96, equ. (5)]

$$F_1\left[p;y_1,y\ ;\frac{1}{2}(p+y_1+y_2+1);\frac{1}{2},\frac{1}{2}\right]$$

$$\frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}+\frac{1}{2}p+\frac{1}{2}y_1+\frac{1}{2}y_2)}{\Gamma(\frac{1}{2}+\frac{1}{2})\Gamma(\frac{1}{2}+\frac{1}{2}y_1+\frac{1}{2}y_2)}, \qquad (4)$$

and Appell and Kampe de Feriet[5, p.22. equ. (4)]

$$F_1[\alpha;\beta,y;\delta;1,1]=\frac{\Gamma(\delta-\alpha-\beta-y)\Gamma(\delta),}{\Gamma(\delta-\alpha)\Gamma(\delta-\beta-y)}, \qquad (5)$$

valid for R $\qquad R(\delta-\alpha-\beta-y)>0.$

2. The summation formula to be proved is

$$F\left[\begin{matrix}p;\ 2\lambda_1+1,\ \lambda_1;\ 2\lambda_2+1,\ \lambda_2;\ \ 1,\ \ 1\\ \frac{1}{2}(p+2\lambda_1+2\lambda_2+3);\ \ 2\lambda_1 2\lambda_2\ \ ;\end{matrix}\right]$$

$$=\frac{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}\frac{1}{2}\ p)\Gamma(\frac{2}{2}+\lambda_1+\lambda_2+\frac{1}{2}p)}{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}+\frac{1}{2}p)\ \Gamma(\frac{3}{2}+\lambda_1+\lambda_2-\frac{1}{2}p)}$$

provided that $R(p)<1$, $R(\lambda\ )>0$ and $R(\lambda_2)>0$

Proof:- To prove (6), we start with the left side of (6).

$$F\left[\begin{matrix}p;\ 2\lambda_1+1,\ \lambda_1;\ 2\lambda_2+1,\lambda_2;\ 1,1\\ \frac{1}{2}(p+2\lambda_1+2\lambda_2+3);\ 2\lambda_1;\ 2\lambda_2;\end{matrix}\right]$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(p)_{m+n}(2\lambda_1+1)_m(\lambda_1)_m(2\lambda_2+1)_n(\lambda_2)_n}{(\frac{1}{2}p+\lambda_1+\lambda_2+\frac{3}{2})_{m+n}(2\lambda_1)_m(2\lambda_2)_n\ m;n;}\ \text{by}\ (2)$$

$$=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(p)_{m+n}(2\lambda_1+1)_m(2\lambda_2+1)_n}{(\frac{1}{2}p+\lambda_1+\lambda_2+\frac{3}{2})_{m+n}\ m!\ n!\ 2^{m+n}}$$

$$\sum_{p=0}^{\frac{1}{2}m}\frac{(-\frac{1}{2}m)_p(\frac{1}{2}-\frac{1}{2}m)_0}{(\lambda_1+\frac{1}{2})_p\ p!}\sum_{q=0}^{\frac{1}{2}n}\frac{(-\frac{1}{2}n)_q(\frac{1}{2}-\frac{1}{2}n)_q}{(\lambda_2+\frac{1}{2})_q\ q!}\ \text{by}\ (3)$$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(p)_{m+n}(2\lambda_1+1)_m(2\lambda_2+1)_n}{(\frac{1}{2}p+\lambda_1+\lambda_2+\frac{3}{2})_{m+n}\ 2^{m+n}}$$

$$\sum_{p=0}^{\frac{1}{2}m}\frac{1}{(m-2p)!\ p!\ (\lambda_1+\frac{1}{2})_p\ 2^{2p}}\sum_{q=0}^{\frac{1}{2}n}\frac{1}{(n-2_q)!\ q!\ (\lambda_2+\frac{1}{2})_q 2^{2q}}$$

$$\sum_{p=0}^{\infty}\sum_{q=0}^{\infty}\frac{(p)_{2p+2q}(2\lambda_1+1)_{2p}(2\lambda_2+1)_{2q}}{(\frac{1}{2}p+\lambda_1+\lambda_2+\frac{3}{2})_{2p+2q}(\lambda_1+\frac{1}{2})_p(\lambda_2+\frac{1}{2})_q\ p!\ q!\ 2^{4p+4q}}$$

$$F\left[\begin{matrix}p+2p+2q;\ 2\lambda_1+1+2p;\ 2\lambda_2+1+2q;\ \frac{1}{2},\frac{1}{2}\\ \frac{1}{2}p+\lambda_1+\lambda_2+\frac{3}{2}+2p;\ +2q;\ -;\ -;\end{matrix}\right]$$

$$=\frac{\Gamma(\frac{1}{2})\Gamma(\frac{3}{2}+\frac{1}{2}p+\lambda_1+\lambda_2)}{\Gamma(\frac{1}{2}+\frac{1}{2}p)\Gamma(\frac{3}{2}+\lambda_1+\lambda_2)}$$

$$F\left[\begin{matrix}\frac{1}{2}p;\ \lambda_1+1;\ \lambda_2+1;\ 1,1\\ \frac{3}{2}+\lambda_1+\lambda_2;\ -;\ -;\end{matrix}\right]\ \text{by}\ (4)$$

$$=\frac{\Gamma(\frac{1}{2})\Gamma(-\frac{1}{2}-\frac{1}{2}p)\Gamma(\frac{3}{2}+\frac{1}{2}p+\lambda_1+\lambda_2)}{\Gamma(\frac{1}{2})\Gamma(\frac{1}{2}+\frac{1}{2}p)\Gamma(\frac{3}{2}+\frac{1}{2}p+\lambda_1+\lambda_2)}\ \text{by}\ (5)$$

This completes the proof of (6).

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