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Development of a methodology to model the dynamic properties of UAVS and high-order control systems

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ABSTRACT

When modeling the dynamic properties of an unmanned aerial vehicle (UAV) and an automatic control system (ACS), it is often necessary to take into account factors such as the lack of rigidity of the aircraft structure, the influence of control and destabilizing factors, which leads to an increase in the order of the model under study. The known numerical and analytical methods do not allow us to obtain general solutions for the variable parameters of the high-order system under study, and at the same time provide the required amount of error and computational costs. A method is developed to model the dynamic properties of UAVs and high-order control systems based on the spectral method, the linear approximation by parts of the input control actions and the spectrum of the system's output signal. An example of using the technique to model the dynamic properties of different orders ACS UAVs (from 1 to 10) with different types of inertia is considered. Based on the analysis of errors in the calculation of the transition process, the effectiveness of the method for analyzing high-order systems is shown based on the required computational costs.

KEY WORDS: unmanned aerial vehicle; UAV; automatic control system; linear approximation by parts; spectral method; transition process.

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Desarrollo de una metodología para modelar las propiedades dinámicas de los UAVS y los sistemas de control de alto orden

RESUMEN

Al modelar las propiedades dinámicas de un vehículo aéreo no tripulado (UAV) y un sistema de control automático (ACS), a menudo es necesario tener en cuenta factores como la falta de rigidez de la estructura de la aeronave, la influencia de las señales de control y los factores desestabilizadores, lo que conduce a un aumento en el orden del modelo en estudio. Los métodos numéricos y analíticos conocidos no nos permiten obtener soluciones generales para los parámetros variables del sistema de alto orden en estudio, y al mismo tiempo proporcionan la cantidad requerida de error y costos computacionales. Se desarrolla un método para modelar las propiedades dinámicas de los UAV y los sistemas de control de alto orden basado en el método espectral, la aproximación lineal por partes de las acciones de control de entrada y el espectro de la señal de salida del sistema. Se considera un ejemplo del uso de la técnica para modelar las propiedades dinámicas de UAV ACS de diferentes órdenes (del 1 al 10) con diferentes tipos de inercia. Con base en el análisis de errores en el cálculo del proceso de transición, se muestra la efectividad del método para analizar sistemas de alto orden por el criterio de los costos computacionales requeridos.

PALABRAS CLAVE: vehículo aéreo no tripulado; UAV; sistema de control automático; aproximación lineal por partes; método espectral; proceso de transición.

Introduction

The design of automatic control systems (ACS) for unmanned aerial vehicles (UAVs) is based on models of the dynamic properties of aircraft in various flight modes. A large number of studies consider the linear dynamics of UAVs (Lebedev & Chernobrovkin, 1973; Moiseev, 2013), and the linearity of their control systems (Byard & McLain, 2015; Kuriki & Namerikawa, 2013). An aircraft of rigid construction with fixed rudders has, like any absolutely solid body, 6 degrees of freedom, and its movement in space is described by a system of 12 differential equations (Lebedev & Chernobrovkin, 1973). The non-rigidity of the aircraft structure or the presence of control actions causes the appearance of additional degrees of freedom, which causes an increase in the order of the UAV models and the control system. At the same time, convenient analytical models describing the dynamics of UAVs and control systems are obtained on the basis of a number of simplifying assumptions for the studied systems of the 4th order maximum (Kuriki & Namerikawa, 2013). Analytical expression for such models can be obtained on the basis of the operator method of Laplace transformations (Dech, 1971). In the simplest cases, the solutions are determined directly from the table of originals and images according to Laplace (Dwight, 1977; Makarov & Mensky, 1978) in more complex cases-using the decomposition theorem of the fractional rational function (Ditkin & Prudnikov, 1961; Shostak, 1972). When the order of the system under study increases, the resulting expressions according to the decomposition theorem become dramatically more complex, especially if the characteristic polynomial of the system has multiple roots. At the same time, lowering the order of the UAV or control system dynamics model may be unacceptable because of the unacceptably high approximation error.

The use of numerical methods for solving differential equations, explicit (Runge-Kutta, Adams) (Lambert, 1991; Godunov & Ryaben'kii, 1962) or implicit for rigid systems (the Bulirsch–Stoer algorithm based on the extrapolation of Richardson, Krank-Nicholson, and others) (Hairer, & Wanner, 1996; Crank, & Nicolson, 1947), allows us to study the dynamics of high-order systems, but does not allow us to obtain analytical solutions that are valid for the modifiable parameters of the model. In addition, when using such methods, as a rule, the computational cost increases significantly with increasing model order.

The aim of this work is to develop a technique for numerical and analytical modeling of UAV dynamics and a high-order control system that is less demanding on computational resources when increasing the model order in comparison with known methods.

1. Generalized model of dynamic properties of UAVs and control systems

Studies have shown that the dynamic properties of UAVs in various flight modes (start, landing, coordinated turn, spiral descent, etc.) and flight control systems (stabilization and guidance systems) are described by a fractional rational transfer function

$$H_{xy}(\mathrm{j}\omega)=\frac{\zeta_{xy}+N_{1xy}M_1(\mathrm{j}\omega)}{1+N_{2xy}M_2(\mathrm{j}\omega)}=\frac{\sum_{i=0}^{I}\alpha_i(\mathrm{j}\omega)^i}{\sum_{i=0}^{I}\beta_i(\mathrm{j}\omega)^i}, \qquad (1)$$

where $j\omega$ is the complex frequency, x – the input control, the response (output parameter) of the system, ζ_{xy} is coefficient, taking values 0 or 1, depending on the specific transfer function, $N_{1,2x,y}$ are coefficients of the forward and backward control circuits (FCS and BCS) for x and y

parameters, I is the order of the system, α_i, β_i are coefficients of the numerator and denominator of the transfer function. A generalized model of the dynamic properties of UAVs and ACS is shown in Fig. 1.

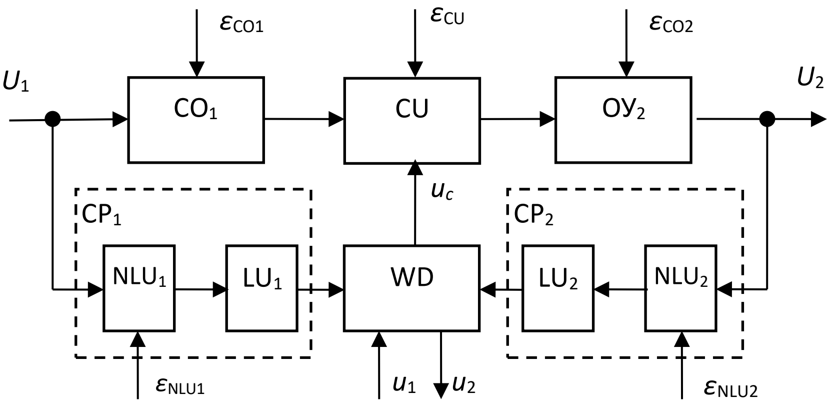

Fig. 1. Generalized model of dynamic properties of UAVs and control system

In the model (Fig. 1) the control object (CO) can be located both at the input of the control unit (CU) – CO_1, and at the output of the CU (CO_2). In addition, the model includes control paths (CP1, 2) and a weight distributor (WD). The CU controls the amplitude and (or) phase of the input action. Each control path in the model is represented by a sequential connection of a nonlinear unit (NLU) and a linear unit (LU). Paths CP_1 and CP_2 implement the principle of forward and backward regulation, respectively. The following designations are accepted in the model: $U_{1,2}$ – main input and output signals of the model, $u_{1,2}$ – auxiliary input and output signals, u_c – control signal, ε - destabilizing factors. For example, the required values of the UAV's angular parameters (roll, pitch, yaw) can be used as input effects, and the achieved values of the corresponding angular velocities can be used as output parameters of the ACS. Nonlinear, inertial and amplifying properties of ACS blocks (angle sensors, servos, etc.) are taken into account at the transfer functions of the corresponding blocks of the generalized model.

2. Methodology for modeling dynamic properties of UAVs and high-order control systems

The use of the spectral method allows us to perform a piecewise linear approximation of the input control actions and the spectrum of the output signal of the system under study, and then obtain the desired expressions of dynamic characteristics by performing the inverse Fourier transform from the output spectrum (Vasilyev et al., 2013; Kurilov et al., 2014). Using a simple approximation with linear segments allows you to get analytical solutions in the same type of record for describing different models of ACS and the control object - UAV. In this case, the computational cost is determined by the number of approximation nodes that must be increased with increasing order of the system under study. However, it should be noted that in terms of computing costs, the proposed approach is less sensitive to the growth of the model order than numerical methods for integrating differential equations.

The conversion of signals in a linear system is described by the expression (Chen, 1998):

$$S_{out}(j\omega) = S_{in}(j\omega) \cdot H_{xy}(j\omega), \tag{2}$$

where $S_{in}(j\omega)$, $S_{out}(j\omega)$ - the spectra of the input control action and the output signal (response) of the system, $H_{xy}(j\omega)$ is the complex transfer function (1).

It is difficult or impossible to obtain an analytical expression of its spectrum $S_{in}(j\omega)$ if the input control action has a complex form. Approximation of the effect based on the switching continuous piecewise function (CPF) (Vasilyev et al., 2013; Kurilov et al., 2014; Vasilyev et al., 2011):

$$q_i(t) = \frac{A_i}{2\Delta_i}\left(\left|t - t_i\right| - \left|t - t_i + \Delta_i\right| + \Delta_i\right), \tag{3}$$

allows us to obtain a compact generalized expression of the spectrum using the direct Fourier transform:

$$S_{in}(j\omega) = \sum_{i=0}^{N-1} \frac{A_i}{\Delta_i \omega^2}\left[e^{-j\omega(t_i+\Delta_i)} - e^{-j\omega t_i}\right], \tag{4}$$

Here it is indicated: i – the number of the current approximation node, t_i – the time in the current node, Δ_i – approximation step, $A_i = x(t_i + \Delta_i) - x(t_i)$ - the difference between the values of the impact parameter at the time $t_i + \Delta_i$ and t_i..

In general, the spectral density of the output parameter of the system can be represented as a real and imaginary part. In (Polivanov, 1972) it is shown that to find the original $y(t) \leftarrow S_{out}(j\omega)$, it is sufficient to use only the real $S_R(\omega)$ or imaginary part $S_I(\omega)$:

$$y_R(t) = \frac{2}{\pi}\int_{0+}^{\infty} S_R(\omega)\cos(\omega t)d\omega + \frac{1}{\pi}\int_{0-}^{0+} S_R\cos(\omega t)d\omega\,; \tag{5}$$

$$y_I(t) = \frac{2}{\pi}\int_{0+}^{\infty} S_I\sin(\omega t)d\omega + \frac{1}{\pi}\int_{0-}^{0+} S_I\sin(\omega t)d\omega\,. \tag{6}$$

These expressions are suitable in some cases for analytical research of dynamic modes. Analytical calculation of transients of real systems using expressions (5) and (6) is not always possible due to the complexity of the resulting expressions of the output signal spectrum. We have to resort to numerical methods, for example, the Philo method for numerical integration of rapidly oscillating functions (Chrysos, 1995; Krylov, 1968). The application of the operator method is also difficult, since the table original for the image of the transition process of a high-order system is usually absent (Dwight, 1977; Makarov & Mensky, 1978). The calculation of the time function by the method of deductions (Hazewinkel, 2001) is generally associated with finding high-order derivatives of several fractional-rational functions. This transformation has to be performed separately for each individual system, which makes research difficult.

Solving integrals in (5) and (6) can be very difficult for a complex transfer function of the system or a complex form of influence. It is convenient to use approximation using switching CPFs (3). We approximate the real $S_R(\omega)$ and imaginary spectrum $S_I(\omega)$ of the output signal by switching CPFs and transform expressions (5) and (6). We first find the dynamic characteristic of the system $y_{R,I}^{(i)}(t)$ in the simplest case, when the spectral density of the output signal has the form of a single switching function.

Based on the approximation of the real output spectrum

$$\begin{aligned} y_R^{(i)}(t) &= -\frac{2}{\pi t}\frac{a_{0i}}{2\Delta_i}\int_{\omega_i-\Delta_i}^{\omega_i+\Delta_i}\sin\omega t d\omega = \frac{2}{\pi t}\frac{a_{0i}}{2\Delta_i}\left[\cos(\omega_i+\Delta_i)t - \cos(\omega_i-\Delta_i)t\right] = \\ &= 2\frac{a_{0i}\omega_i}{\pi}\frac{\sin\omega_i^* t}{\omega_i^* t}\frac{\sin\Delta_i^* t}{\Delta_i^* t}, \end{aligned} \tag{7}$$

where $a_{0i} = S_R(\omega_i) - S_R(\omega_{i+1})$ is coefficient of the CPF approximating $S_R(\omega)$ at the current segment $[\omega_i;\omega_{i+1}]$, $\Delta_i^* = \Delta_i/2$, $\omega_i^* = \omega_i + \Delta_i/2$ is the central frequency of the inclined side of the i-th switching CPF.

Based on an approximation of the imaginary output spectrum

$$y_I^{(i)}(t)=\frac{2}{\pi t}S_I(\delta_\omega)+\frac{2}{\pi t}\frac{\mathrm{b}_{0\mathrm{i}}}{2\Delta_i}\int_{\omega_i-\Delta_i}^{\omega_i+\Delta_i}\cos\omega t d\omega=\frac{2}{\pi t}S_{\textit{вых}2}(\delta_\omega)+$$
$$+\frac{b_{0\mathrm{i}}}{\pi t^2\Delta_i}\left[\sin(\omega_i+\Delta_i)t-\sin(\omega_i-\Delta_i)t\right]= \quad (8)$$
$$=\frac{2}{\pi t}S_I(\delta_\omega)+2\frac{b_{0\mathrm{i}}\omega_i}{\pi}\frac{\cos\omega_i^* t}{\omega_i^* t}\frac{\sin\Delta_i^* t}{\Delta_i^* t},$$

where $b_{0i}=S_I(\omega_\mathrm{i})-S_I(\omega_{\mathrm{i}+1})$ is the coefficient of the CPF approximating $S_I(\omega)$ in the i-th segment. We apply an approximation based on switching CPF at N nodes for the real and imaginary spectrum of the output parameter of the system:

$$S_R(\omega)=\sum_{i=0}^{N-1}a_i(\omega),\quad S_I(\omega)=\sum_{i=0}^{N-1}b_i(\omega). \quad (9)$$

Dynamic characteristic based on the real output spectrum has the form

$$y_R(t)=\frac{2}{\pi}\sum_{i=0}^{N-1}a_{0i}\omega_i\frac{\sin\omega_i^* t}{\omega_i^* t}\frac{\sin\Delta_i^* t}{\Delta_i^* t}. \quad (10)$$

Dynamic characteristic based on the imaginary output spectrum is obtained by summing (8) over N approximation nodes:

$$y_I(t)=\frac{2}{\pi t}S_\mathrm{I}(\delta_\omega)+\frac{2}{\pi}\sum_{i=0}^{N-1}b_{0i}\omega_i\frac{\cos\omega_i^* t}{\omega_i^* t}\frac{\sin\Delta_i^* t}{\Delta_i^* t},\quad \delta_\omega\to 0. \quad (11)$$

The use of expressions (10) and (11) is incorrect if the output spectrum contains a 2nd-order gap at a certain frequency: $S_R(j\omega_k)\to\infty$. To avoid such situations, we present the complex transfer function of the converter in the form $H(j\omega)=H(j\omega_k)+\left[H(j\omega)-H(j\omega_k)\right]$ and denote $H^*(j\omega)=H(j\omega)-H(j\omega_k)$. Then

$S_{out}(j\omega)=S_{out0}(j\omega)+S^*_{out}(j\omega)=S_{in}(j\omega)\cdot H(j\omega_k)+S_{in}(j\omega)\cdot H^*(j\omega)$,

and for the time dependence of the signal, we get

$$y_R(t)=x(t)\cdot H(j\omega_k)+\frac{2}{\pi}\sum_{i=0}^{N-1}a_{0i}^*\omega_i\frac{\sin\omega_i^* t}{\omega_i^* t}\frac{\sin\Delta_i^* t}{\Delta_i^* t} \quad (12)$$

$$y_I(t)=x(t)\cdot H(j\omega_k)+\frac{2}{\pi}\sum_{i=0}^{N-1}b_{0i}^*\omega_i\frac{\cos\omega_i^*}{\omega_i^*}\frac{\sin\Delta_i^* t}{\Delta_i^* t}, \quad (13)$$

where $a_{0i}^*=S_\mathrm{R}^*(\omega_\mathrm{i})-S_R^*(\omega_{\mathrm{i}+1})$, $b_{0i}^*=S_I^*(\omega_\mathrm{i})-S_I^*(\omega_{\mathrm{i}+1})$ - coefficient of the i-th switching CPF.

3. Modeling the dynamic properties of UAV ACS

The transient characteristics of ACS with combined control and unit control coefficients for forward (FCS) and backward control (BCS) (N_1=N_2=1) calculated using expressions (12) and (13) are shown in Fig. 2-4. A single step function (Heaviside function) was adopted as the input control action U1. The characteristics of a system with three types of filters of different order in the FCS and BCS circuits are shown: with low-pass filters (LPF, Fig. 2), high-pass filters (HPF, Fig. 3) and bandpass filters (BPF, Fig. 4). In each case, two identical filters were used in the FCS and in the BCS. In the examples considered, the LPF and HPF have the 1st, 2nd, 3rd, and 5th order; the BPF is the 2th, 4th, 6th, and 10th order. High-order filters are formed by a series of 1st-order filters of the corresponding type. The ratio of the time constants of the LPF and HPF links in the BPF in Fig. 4 is assumed to be γ=1.

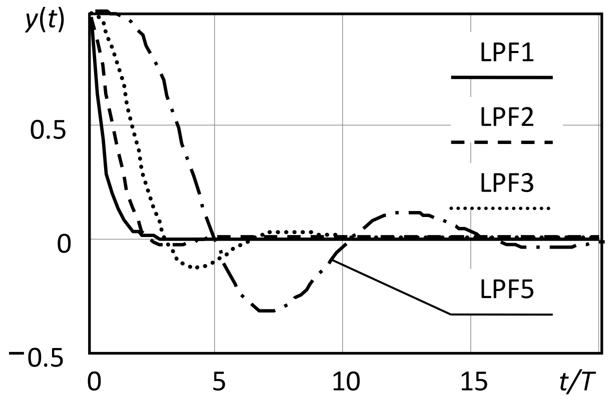

Fig. 2. Transient characteristics of the UAV ACS with low-pass filters of different orders

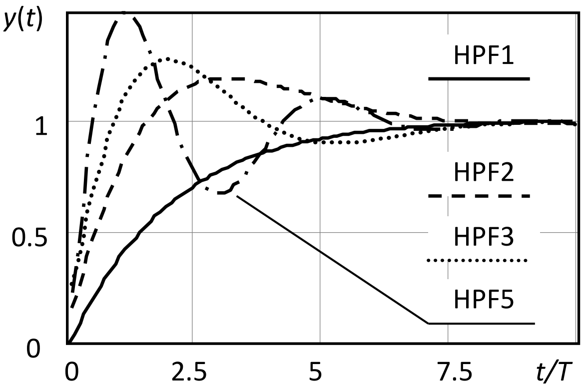

Fig. 3. Transient characteristics of the UAV ACS with high-pass filters of different orders

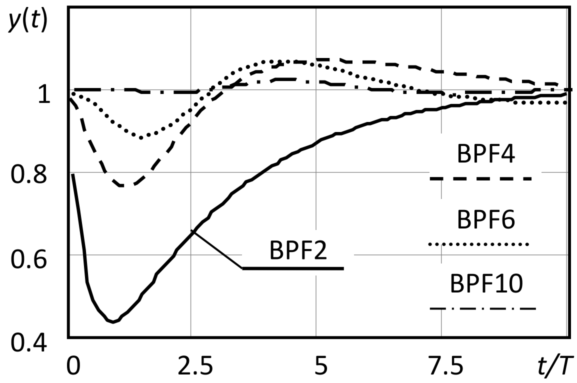

Fig. 4. Transient characteristics of the UAV ACS with bandpass filters of different orders

The study has shown that for transients of different types (aperiodic, oscillatory), for different order of the system and type of inertia, the mean square error of the calculation was from $5 \cdot 10^{-4}$ to $9 \cdot 10^{-4}$ at N=100 approximation nodes.

Conclusions

The task of modeling and analyzing the dynamic properties of the UAV and control system is significantly complicated by factors such as the non-rigidity of the aircraft structure, the influence of control signals and destabilizing factors, which leads to an increase in the order of the model under study. Known numerical and analytical methods do not allow us to obtain general solutions for the variable parameters of the high-order system under study, and at the same time provide the required error value. In addition, when using numerical methods for integrating differential equations, as a rule, computational costs increase significantly with increasing model order.

A generalized model of the dynamic properties of UAVs and high-order ACS is proposed, which allows for the simulation of dynamics to take into account the influence of control signals, destabilizing factors, and characteristics of sensors and actuators in the transfer functions of the corresponding blocks of the generalized model. A method for modeling the dynamic properties of UAVs and high-order control systems has been developed. The use of the spectral method, as well as piecewise linear approximation of the input control actions and the spectrum of the output signal of the system, allowed us to obtain similar analytical expressions for systems of different orders. Based on the developed method, the dynamic properties of UAV ACS of different orders (from the 1st to the 10th) with different types of inertia were simulated. The study has shown that for transients of different types (aperiodic, oscillatory), for different order of the system and type of inertia, the mean square error of the calculation was from 5·10−4 to 9·10−4 at N=100 approximation nodes. Thus, with an increase in the order of the system under study, the number of approximation nodes (and, consequently, computational costs) does not need to be increased, and acceptable accuracy is provided for a fixed N. Thus, the efficiency of the method for analyzing high-order systems based on the criterion of required computational costs is shown.

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