opción

Revista de Antropología, Ciencias de la Comunicación y de la Información, Filosofía, Lingüística y Semiótica, Problemas del Desarrollo, la Ciencia y la Tecnología

Año 36, 2020, Especial N° 27

Revista de Ciencias Humanas y Sociales

ISSN 1012-1587/ ISSNe: 2477-9385

Depósito Legal pp 198402ZU45

Universidad del Zulia

Facultad Experimental de Ciencias

Departamento de Ciencias Humanas

Maracaibo - Venezuela

Opción, Año 36, Especial No.27(2020):1543-1561
ISSN 1012-1587/ISSNe: 2477-935

The different system with preventive maintenance and repair

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Abstract

The study aims to investigate different systems with preventive maintenance and repair. The system is analyzed by the semi-Markov process technique, to solving the equations by using Laplace transformation for integral equations. As a result, it is better to use preventive maintenance under the system I because the mean lifetime of the system under this system is greater than the mean lifetimes for the system under system III and II. In conclusion, the meantime for the system with repair and preventive maintenance is greater than the meantime for the system with a repair only can be proved.

Keywords: Preventive, Maintenance, Repair, Lifetime, Operative.

El sistema diferente con mantenimiento preventivo y reparación

Resumen

El estudio tiene como objetivo investigar diferentes sistemas con mantenimiento preventivo y reparación. El sistema se analiza mediante la técnica de proceso de semi-Markov, para resolver las ecuaciones mediante el uso de la transformación de Laplace para ecuaciones integrales. Como resultado, es mejor utilizar el mantenimiento preventivo bajo el sistema I porque la vida media del sistema bajo este sistema es mayor que la vida media del sistema bajo

Recibido: 20-12-2019 •Aceptado: 20-02-2020

los sistemas III y II. En conclusión, el tiempo para el sistema con reparación y mantenimiento preventivo es mayor que el tiempo para el sistema con reparación solo se puede probar.

Palabras clave: Preventivo, Mantenimiento, Reparación, Vida útil, Operativo.

1. INTRODUCTION

Three different systems for preventive maintenance has been considered for the standby duplex system with preventive maintenance and repair. NAKAGWA & OSAKI (1975) have mentioned that there were some faults in the investigation in MOKADDIS, KHALIL & HANAA (2016) of the standby duplex system under system III. System III has been investigated, system III giving its final correct form. The mean lifetime of the duplex system under each system has been given. The comparing between these three systems, numerically and theoretically, has been found that the first system is the optimal system.

The three different preventive maintenance systems have been considered to be as follows: Under preventive maintenance type I system, the operative unit undergoes inspection, when its inspection time is due, only if the other unit is in a standby state. The other unit is switched on the continue the job, but if the inspection time of the operative unit comes while the other unit is under repair or inspection the inspection of the operative unit is not made even after completing the repair or the inspection of the other unit. Under the preventive maintenance type II system, the operative unit undergoes inspection, when its inspection time comes, regardless of the state of the other unit (MOKADDIS, KHALIL & HANAA, 2016).

Under type III system, the operative unit undergoes inspection. Its inspection time is due, only if the other unit is in standby stat; the other unit is switched on to continue the job, but if the inspection time of the operative unit is due while the other unit is under repair or inspection, the operative unit goes to inspection after the repair of the failed unit or after the inspection of the unit under inspection is completed (CHANDRASEKHAR, NATARJAN & YADAVALLI, 2004).

The difference between the system I and system III is that if the inspection time of the operative unit is due when the other unit is under repair or inspection then, under system III the operative unit undergoes inspection after the completion of the repair or the inspection of the other unit, but under the system I the operative unit does not go the inspection and continues operating until its failure. To study the duplication system under each of these systems several assumptions are imposed on the system. These assumptions are:

1. The lifetime of each unit is a random variable and has an arbitrary distribution function F (.).

2. The repair time of each unit is a random variable and has an arbitrary distribution function G (.).

3. The inspection time of each unit is a random variable and has an arbitrary distribution function U (.).

4. The time from inspection beginning to inspection completion is a random variable and has an arbitrary distribution function V (.).

5. The repair or the inspection of the unit completely restores all the initial properties of the unit.

6. The switch over time, from failure to repair, from repair completion to standby state from the standby state to operative state, and from inspection state to operative or standby state, of each unit, are all assumed to be negligible.

7. The time distribution G (.) is ≤ the time distribution V (.).

8. As seen as the main unit fails, the standby unit immediately assumes the lead of the failed unit, the repair of the failed unit or the inspection begins immediately.

To obtain the reliability function and the mean lifetime of the system under each system, we use the following terminology:

R (t): is the reliability function of the system at time t, where initially at t= 0 the main unit and the standby unit are completely new, i.e. the main unit starts to do the job and the other is standby.

R1¬(t): is the reliability function of the system at time t, where initially at t= 0, one unit is under repair and the other unit is operating.

R2¬(t): is the reliability function of the system at time t, where initially at t= 0, one unit is under inspection and the other is operating (BARLOW & PROSCHAN, 1965).

2. METHODOLOGY

The integral equations for the reliability distributions $R\,(t\,), R_1\,(t\,)\;and\;\;R_2\,(t\,)$. Under type I system can be obtained as follows:

$$R(t) = \overline{F}(t)\overline{U}(t) + \int_0^t \overline{U}(x) R_1(t-x)\,d\,F(x) + \int_0^t \overline{F}(x) R_2(t-x)\,dU(x) \qquad (3.1)$$

$$\begin{aligned} R_1(t) = {} & \overline{F}(t)\overline{U}(t) + \overline{F}(t)\overline{G}(t)U(t) + \overline{F}(t)\int_0^t U(x)\,dG(x) + \\ & \int_0^t \overline{U}(x)G(x)R_1(t-x)\,dF(x) + \int_0^t \overline{F}(x)G(x)R_2(t-x)\,dU(x) + \\ & \int_0^t R_1(t-x)\int_0^x U(y)\,dG(y)\,dF(x) \end{aligned} \qquad (3.2)$$

$$R_2(t) = \bar{F}(t)\bar{U}(t) + \bar{F}(t)\bar{V}(t)U(t) + \bar{F}(t)\int_0^t U(x)dV(x) + \int_0^t \bar{U}(x)V(x)R_1(t-x)dF(x) + \int_0^t \bar{F}(x)V(x)R_2(t-x)dU(x) + \int_0^t R_1(t-x)\int_0^x U(y)dV(y)dF(x) \quad (3.3)$$

For equation (3.2), the third term is that: the operative unit has not failed till time t and its inspection time comes at time x (x<t) before the repair completion of the failed unit, the repair of the failed unit ends before the time t; therefore the inspection of the operative unit is not done till the time t. the probability of this event is

$$\bar{F}(t)\int_0^t U(x)\,dG(x)\cdot$$

The last term of equation (3.2)is that; the operative unit is called for inspection at time y (y< x< t) before the repair completion of the failed unit and so on, the inspection of the operative unit is not done and is left to operate, the repair of the failed unit ends before time x and the operative unit fails at time x, and then the system with one failed unit at time x works the time (t-x) without failure (MOKADDIS, AYED & AL-HAJERI, 2013). The probability of this event is

$$\int_0^t R_1(t-x)\int_0^x U(y)\,dG(y)\,dF(x)\cdot$$

The explanation of the other term in (3.2) is given in system I. For the analysis of equation (3.3): we have the following. The third term means that: the operative unit has not failed till the time t and its inspection time comes at time x (x<t) before the inspection completion of the maintained unit ends before time t, therefore, the inspection of the operative unit is not done (GENDENKO, BEDYAEV & SOLEVYEV, 1969).

The last term means that: the operative unit is called for inspection at time y(y< x < t) before the inspection completion of the maintained unit and therefore the inspection of the operative unit is not done and the unit is left to operate, the inspection of the maintained unit ends before the time x and the operative unit fails at time x and then the system with one failed unit at time x works the time (t-x) without failure. The analysis of the other terms in equation (3.3) is given in system III above (GOEL & GUPTA, 1984).

To solve the integral equations (3.1), (3.2), and (3.3) we introduce the Laplace transforms:

$$\hat{b}(s) = \int_0^\infty e^{-sy}\, \overline{F}(t) \int_0^t U(x)\,dG(x)\,dt \,,$$

$$\hat{c}(s) = \int_0^\infty e^{-st}\, \overline{F}(t) \int_0^t U(x)\,dV(x)\,dt \,,$$

$$\alpha_1(s) = \int_0^\infty e^{-st} \int_0^t U(y)\,dG(y)\,dF(t) \,,$$

$$\alpha_2(s)=\int_0^\infty e^{-st}\int_0^t U(y)dV(y)dF(t) \quad . \qquad (3.4)$$

Taking the Laplace transform (3.1), (3.2) and (3.3), applying (3.4), we get,

$$R^*(s)=a(s)+d_1(s)R_1^*(s)+d_2(s)R_2^*(s) \qquad (3.5)$$

$$\begin{aligned}R_1^*(s)&=a(s)+b(s)+\hat{b}(s)+b_1(s)R_1^*(s)+c_1(s)R_2^*(s)+\alpha_1(s)R_1^*(s)\\&=a_1(s)+\left[b_1(s)+\alpha_1(s)\right]R_1^*(s)+c_1(s)R_2^*(s)\end{aligned} \qquad (3.6)$$

$$\begin{aligned}R_2^*(s)&=a(s)+c(s)+\hat{c}(s)+b_2(s)R_1^*(s)+c_2(s)R_2^*(s)+\alpha_2(s)R_1^*(s)\\&=a_2(s)+\left[b_2(s)+\alpha_2(s)\right]R_1^*(s)+c_2(s)R_2^*(s)\end{aligned} \qquad (3.7)$$

where;

$$a_1(s)=a(s)+b(s)+\hat{b}(s),\ a_2(s)=a(s)+c(s)+\hat{c}(s) .$$

Solving (3.5), (3.6) and (3.7) in the three unknowns $R_1^*(s), R_1^*(s), R_2^*(s)$, we get

$$R_1^*(s)=\{a_1(s)[1-c_2(s)]+a_2(s)c_1(s)\}/D_3(s)$$

$$R_2^*(s)=\{a_1(s)[\alpha_2(s)+b_2(s)]+a_2(s)[1-\alpha_1(s)-b_1(s)]\}/D_3(s)$$

$$\begin{aligned}R^*(s)&=a(s)+\{a_1(s)\{d_1(s)[1-c_2(s)]+d_2(s)[\alpha_2(s)+b_2(s)]\}+\\&a_2(s)\{d_1(s)c_1(s)+d_2(s)[1-\alpha_1(s)-b_1(s)]\}\}/D_3(s)\end{aligned} \qquad (3.8)$$

where;

$$D_3(s)=\left[1-\alpha_1(s)-b_1(s)\right]\left[1-c_2(s)\right]-\left[\alpha_2(s)+b_2(s)\right]c_1(s)$$

The mean lifetime of the system is given by

$$T_3=R^*(0)=\int_0^\infty R(t)dt$$

$$a(0)+\left\{a_1(0)\left[d_1(0)\left(1-c_2(0)\right)+d_2(0)\left(\alpha_2(0)+b_2(0)\right)\right]+\right.$$
$$\left.+a_2(0)\left[d_1(0)c_1(0)+d_2(0)\left(1-\alpha_1(0)-b_1(0)\right)\right]\right\}\Big/D_3 \qquad (3.9)$$

If there is no preventive maintenance but only repair, we have

$$a(0)=\int_0^\infty \overline{F}(t)dt\ ,\ b_1(0)=\int_0^\infty G(t)dF(t)$$

$$d_1(0)=1,\ b(0)=c(0)=b_2(0)=c_1(0)=e_2(0)=d_2(0)=0$$

$$\hat{b}(0)=\hat{c}(0)=\alpha_1(0)=\alpha_2(0)=0.$$

So that the mean lifetime of the duplication system with a repair only is given by

$$T_1=T_2=T_3=T'=a(0)+\frac{a(0)}{1-b_1(0)} \qquad (3.10)$$

where;

$$a(0)=\int_0^\infty \overline{F}(t)\,dt,\ b_1(0)=\int_0^\infty \overline{G}(t)\,dF(t)\qquad .$$

This result is the result obtained in MOKADDIS, KHALIL & HANAA (2016) when the duplication system is with a repair only.

Following the same analysis in the type III system, we obtain the following three integral equations for the reliability distribution $R(t)$, $R_1(t)$ *and* $R_2(t)$

$$R(t)=\overline{F}(t)\,\overline{U}(t)+\int_0^t \overline{U}(x)\,R_1(t-x)\,dF(x)+\int_0^t \overline{F}(x)\,R_2(t-x)\,dU(x)$$

$$R_1(t)=\overline{F}(t)\,\overline{U}(t)+\int_0^t G(x)\,\overline{U}(x)\,R_1(t-x)\,dF(x)+\int_0^t G(x)\,\overline{F}(x)\,R_2(t-x)\,dU(x)$$

$$R_2(t)=\overline{F}(t)\,\overline{U}(t)+\int_0^t V(x)\,\overline{U}(x)\,R_1(t-x)\,dF(x)+\int_0^t V(x)\,\overline{F}(x)\,R_2(t-x)\,dU(x). \qquad (4.1)$$

The terms in equations (4.1) are all established and discussed in the previous section.

4. CONCLUSION

We conclude that it is better to use preventive maintenance under the system I because the mean lifetime of the system under this system is greater than the mean lifetimes for the system under system III and II.

THEOREM 2: The meantime for the system with repair and preventive maintenance T_3 is greater than T_2 the meantime for the system with a repair only under the assumption that the failure rate $r(t)$ of the failure time distribution is strictly increasing and that we adopt a suitable inspection interval T .

PROOF:

$$T_3 = a_0\left[1+\frac{1+F(T)P(T)\langle G(T)-V(T)\rangle+P(T)\int_0^T\big(V(T)-G(t)\big)dF(t)}{[1-\int_0^T G(T)dF(t)][1-V(T)P(T)]-P(T)G(T)\int_0^T V(t)dF(t)]}\right]$$

$$T_2 = a_\infty\left[1+\frac{1}{1-\int_0^\infty G(t)dF(t)}\right].$$

We assume that

$$F \equiv F(T) \quad , \quad P \equiv P(T) = 1-F(T) \quad , \quad V \equiv V(T) \quad ,$$

$$G \equiv G(T) \quad , \quad V_T = \int_0^T V(t)dF(t) \quad ,$$

$$G_T = \int_0^T G(t)dF(t) \quad , \quad G_\infty = \int_0^\infty G(t)dF(t),$$

Then

$$T_3 - T_2 = \frac{\left[(a_0 - a_\infty)(1-G_\infty) - a_\infty\right]\left[(1-G_T)(1-VP) - PGV_T\right]}{(1-G_\infty)\left[(1-G_T)(1-VP) - PGV_T\right]} + \frac{a_0(1-G_\infty)\left[1 + FP(G-V) + P(V_T - G_T\right]}{(1-G_\infty)\left[(1-G_T)(1-VP) - PGV_T\right]}.$$

We shall show that $T_3 - T_2$ is positive.

For the denominator we have $(1-G_\infty) > 0$ and

$$\begin{aligned}(1-G_T)(1-VP)PGV_T &> (1-V_T)(1-VP) - PVV_T \\ &= 1 - V_T - VF > 1 - F - VP \\ &= P - VP = P(1-V) > 0\end{aligned}$$

Since

$$V_T = \int_0^T V(t)dF(t) < \int_0^T dF(t) = F$$

for the numerator, let

$$A(T) = \left[(a_0 - a_\infty)(1-G_\infty) - a_\infty\right]\left[(1-G_T)(1-VP) - PGV_T\right] + a_0(1-G_\infty)\left[1 + FP(G-V) + P(V_T - G_T\right].$$

then

$$
\begin{aligned}
\frac{dA(T)}{dT} = & \left[a_0'(1-G_\infty)\right]\left[(1-G_T)(1-VP)-PGV_T\right] \\
& +\left[(a_0-a_\infty)(1-G_\infty)-a_\infty\right]\left[-G_T'(1-VP)\right. \\
& \left.-(1-G_T)(PV'+VP')-V_T(PG'+GP')-V_T'PG\right] \\
& +a_0'(1-G_\infty)\left[1+FP(G-V)+P(V_T-G_T)\right] \\
& +a_0(1-G_\infty)\left[FP(G'-V')+(G-V)(FP'+F'P)\right. \\
& \left.+P'(V_T-G_T)+(P(V_T'-G_T')\right]
\end{aligned}
$$

Since $a_0 = \int_0^T (1-F(t))dt$, then

$$a_0' = \frac{da_0}{dT} = 1 - F(T)dt$$

$$a_0' = \frac{da_0}{dT} = 1 - F(T) = P(T) = P \cdot$$

Also $G_T' = \frac{d}{dT}\int_0^T G(t)f(t)dt = G(T)f(T) = Gf$,

$V_T' = Vf$, $P' = -f$, and $F' = f$

where $f = f(T)$ is the probability density function of the failure time.

$$\frac{dA(T)}{dT} = P(1-G_{\infty})\left[(1-G_T)(1-VP) - PGV_T\right]$$
$$+\left[(a_0 - a_{\infty})(1-G_{\infty}) - a_{\infty}\right]\left[f\{V(1-G_T) - G(1-V_T)\}\right]$$
$$+P(1-G_{\infty})\left[1+P(1-P)(G-V)+P(V_T - G_T)\right]$$
$$-a_0(1-G_{\infty})\left[P(1-P)\right](G'-V') + f(V - G - V_T + G_T) - Pf(V - G)$$

Since the failure note function r is equal to F/P, then

$$\frac{dA(T)}{dT} = P(1-G_{\infty})\left[(1-G_T)(1-VP) - PGV_T\right.$$
$$+1-P(1-P)(V-G)+P(V_T - G_T)$$
$$\left.+(a_{\infty} - a_0)\{V'(1-G_T) + G'V_T\}\right] + Pa_{\infty}\{V'(1-G_T) + G'V_T\}$$
$$-a_0 P(1-G_{\infty})(V'-G')(1-P)$$
$$-rP\{a_{\infty} + (a_{\infty} - a_0)(1-G_{\infty})\}\{V(1-G_T) - G(1-V_T)\}$$
$$-rPa_0(1-G_{\infty})(G - V + V_T - G_T - rP^2 a_0(1-G_{\infty})(V-G)$$

Now we can show that there exists a T^* such that $\frac{dA(T)}{dT} = 0$

where r is an increasing function, that is

$$
\begin{aligned}
r(T^{*}) \equiv (1-G_{\infty})[(1-G_{T*})(1-PV) - PGV_{T*} + 1 - P(1-P)(V-G) \\
+ P(V_{T} - G_{T*}) + (a_{\infty} - a_{0})\{V'(1-G_{T*})G'V_{T}'\}] \\
+ a_{\infty}\{V'(1-G_{T*}) + G'V_{T*}\} - a_{0}(1-G_{\infty})(V' - G')(1-P) \\
/\{a_{\infty} + (a_{\infty} - a_{0})(1-G_{\infty})\}\{V(1-G_{T*}) - G(1-V_{T*})\} \\
+ a_{0}(1-G_{\infty})(G - V + V_{T*} - G_{T*}) + Pa_{0}(1-G_{\infty})(V - G)
\end{aligned}
$$

For the numerator we have

$(1-G_{\infty}) > 0$ and

$$
\begin{aligned}
&[(1-G_{T*})(1-PV) - PGV_{T*} + 1 - P(1-P)(V-G) + P(V_{T*} - G_{T*}) \\
&+ (a_{\infty} - a_{0})\{V'(1-G_{T*}) + G'V_{T*}] > 0
\end{aligned}
$$

since

$(1-G_{T*})(1-PV) - PGV_{T*} > 0$,

$1 - P(1-P)(V-G) > 0$,

$P(V_{T*} - G_{T*}) > 0$,

$(a_{\infty} - a_{0}) > 0$,

and $V'(1-G_{T*})+G'V_{T*}>0$. Also

$$a_\infty\{V'(1-G_{T*})+G'V_{T*}\}-a_0(1-G_\infty)(V'-G')(1-P)>0$$

since $a_\infty > a_0$, and

$$\begin{aligned}&V'(1-G_{T*})+G'V_{T*}-(1-G_\infty)(V'-G')(1-P)\\ &=V'(1-G_{T*})-V'(1-G_\infty)(1-P)+G'V_{T*}+G'(1-G_\infty)(1-P)>0\end{aligned},$$

since $1-G_{T*}>1-G_\infty$ and $1-P<1$.

For the denominator we have

$$(a_\infty-a_0)(1-G_\infty)\{v(1-G_{T*})-G(1-V_{T*})\}>0,$$

Since $a_\infty > a_0$ $1-G_\infty>0, V>G$, $1-G_{T*}>1-V_T$.

And $a_0(1-G_\infty)(V-G)>0$.

Also

$$a_\infty\{V(1-G_{T*})-G(1-V_{T*})\}-a_0(1-G_\infty)(V-G)$$
$$=V\{a_\infty(1-G_{T*})-a(1-G_\infty)\}-G(a_\infty(1-V_{T*})-a_0(1-G_\infty)\}>0 \quad ,$$

since $a_\infty > a_0 \qquad (1-G_{T*})>(1-G_\infty)$.

We have $a_\infty(1-G_{T*})-a_0(1-G_\infty)>0$

hence if $a_\infty(1-V_{T*})-a_0(1-G_\infty)<0$

then

$$V\{a_\infty(1-G_{T*})-a_0(1-G_\infty)\}-G\{a_\infty(1-V_{T*})-a_0(1-G_\infty)\}>0$$

and if $a_\infty(1-V_{T*})-a_0(1-G_\infty)>0$

then

$$V\{a_\infty(1-G_{T*})-a_0(1-G_\infty)\}-G\{a_\infty(1-V_{T*})-a_0(1-G_\infty)\}>0$$

since $V>G$ and $1-G_{T*}>1-V_{T*}$.

It follows that $r(T^*)$ is positive.

Since $A(0)<0$, $A(\infty)=0$ and $A(r)$ is a unimodal function,

These exist a $T' < \infty$ such that $A(T')=0$, i.e. these exists T' such that

$$\begin{aligned}&a_0(1-G_\infty)[(1-G_T')(1-VP)-PGV_{T'}+1-P(1-P)(V-G)+P(V_{T'}-G_{T'})]\\&=a_\infty(2-G_\infty)\left[(1-G_T)(1-VP)-PGV_T\right]\end{aligned}.$$

Hence if we choose a $T > T'$, we have $A(T)>0$.

Thus T_3-T_2 is positive if we choose $T > T'$. This proves the theorem.

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opción
Revista de Ciencias Humanas y Sociales

Año 36, Especial N° 27 (2020)

Esta revista fue editada en formato digital por el personal de la Oficina de Publicaciones Científicas de la Facultad Experimental de Ciencias, Universidad del Zulia.
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