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A bosonic realisation of the 1-D parabose oscillator

Ramón J. Cova*

Departamento de Física, Facultad Experimental de Ciencias. Universidad del Zulia, Apartado 15332, Maracaibo 4005-A, Venezuela.

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Abstract

In the Fock space of two Bose operators all the representations of the 1-D Parabose oscillator are realised. States of $(p+2n-1)$ bosons are shown to stand for states of n parabosons of order p.

Key words: Bose; parastatistics; quantum oscillator.

Una realización bosónica del oscilador de parabose en 1-D

Resumen

Se construyen, en el espacio de Fock de dos operadores de Bose, todas las representaciones del oscilador 1-D de parabose. Se demuestra que estados de n bosones equivalen a estados de $(p+2n-1)$ parabosones de orden p.

Palabras clave: Bose, paraestadística, oscilador cuántico.

Introduction

Parastatistics is a generalisation of both Bose and Fermi statistics, introduced by Green (1) using trilinear relations in lieu of the bilinear relations obeyed by Fermi and Bose systems. The Fock space of an order *p* (*p* any positive integer) para-particle system is characterised by

$$[[a_k^+, a_l]_\pm, a_r]_- = -2\delta_{kr}a_l \qquad [1]$$

$$[[a_k, a_l]_\pm, a_r]_- = 0 \qquad [2]$$

$$[[a_k, a_l]_\pm, a_r^+]_- = 2\delta_{kr}a_l \pm 2\delta_{kr}a_l \qquad [3]$$

where the upper (lower) signs refer to parabosons (parafermions). Equation [3] stems from [1]-[2] with the help of Jacobi´s identity

$$[[d,e]_\pm, f]_- + [[f,d]_\pm, e]_- + [[e,f]_\pm, d]_- = 0$$

We are using the notation $[a_k, a_l]_\pm = a_k a_l \pm a_l a_k$. The para-algebra is supplemented by the Greenberg-Messiah condition on the vacuum (2)

$$a_k a_l^+ |0> = \delta_{kl} p |0>. \qquad [4]$$

The standard Bose-Einstein and Fermi-Dirac statistics correspond to the special case *p =1*.

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* Autor para la correspondencia. E-mail: rjcova@sdf.lonestar.org.

Originally intended for elementary particles [3] in connection with quarks,

today parastatitics also finds a sound setting in condensed matter through quasiparticles and fractional statistics (4). In the literature the term ¨parastatistics¨ is also used re phase changes ≠ ±1 in wave functions when two objects, eg, kinks, swap positions (5).

Using the bosonisation techniques of Kálnay and others (6) the present paper reproduces the parabose algebra in terms of two Bose operators. The Bose description of fermions and paraparticles is an important tool with applications in particle theory, solitons and other areas of theorical physics (7, 8, 9, 10).

Bosonisation

Consider two Bose operators b_1, b_2 and their adjoints acting on a Fock space with basis $|m_1, m_2>, m_1, m_2 = 0,1,2,...,$ obeying the well known commutation relations

$$[b_i, b_j^+]_- = \delta_{ij}, [b_i, b_j]_- = [b_i^+, b_j^+]_- = 0 \qquad [5]$$

where i, j = 1,2. also consider vectors of the form

$$|n> = |p+n-1, n>, \qquad [6]$$

$n = 0,1,2,\ldots \; p = 1,2\ldots$

Introduce the annihilation and creation operators (11)

$$B = gb_1b_2, \quad B^+ = b_2^+ b_1^+ g, \qquad [7]$$

where

$$g \equiv g(m_1, m_2) = g^+(m_1, m_2) \qquad [8]$$

is a real function of the Bose number operator

$$m_j = b_j^+ b_j, \; j = 1,2. \qquad [9]$$

If the function is chosen to have the spectrum

$$g(p+n-1, n) = \begin{cases} (n+1)^{-1/2} \, neven \\ (n+p)^{-1/2} \, n\, odd \end{cases}, \qquad [10]$$

then application of the operators (7) onto the kets (6) entails

$$B|n> \begin{Bmatrix} \sqrt{n} \\ \sqrt{p+n-1} \end{Bmatrix} |n+1>$$

[11]

and

$$B^+|n> = \begin{Bmatrix} \sqrt{p+n} \\ \sqrt{n+1} \end{Bmatrix} |n+1> \qquad [12]$$

where the upper (lower) expression corresponds to even (odd). Equations [11]-[12] can be compactly written in matrix form:

$$B_{n,n}+1 = B^+_{n+1,n} = \begin{cases} \sqrt{p+n} \\ \sqrt{n+1} \end{cases}, \qquad [13]$$

which we readily recognise as the Fock representation of the one dimensional parabose oscillator. Indeed, it is straightforward to verify the parabose algebra [1]-[3] (upper signs)

$$[[B^+, B]_+, B]_- = -2B$$
$$[[B, B]_+,]_- = 0$$
$$[[B, B]_+, B^+]_- = 4B \qquad [14]$$

as well as the occupation number eigen-equation for parabosons

$$N|n> = n|n>, \qquad [15]$$

where N is the paraboson number operator

$$N \equiv \frac{1}{2}\left([B^+, B]_+ - p\right). \qquad [16]$$

For n = 0 the expressions (15)-(16) give the Greenberg-Messiah condition (4)

$$B^+B|0> = p|0>, \qquad [17]$$

thus unveiling p the order of parastatistics or paraquantisation.

Remarkably, from the kets (6) and the parabose number operator (16) we see that $(p+2n-1)$ particle Bose states can be regarded as n particle parabose states of order p. All the irreducible Fock representations of the algebra (14) are parametrised by p, which is fixed by (17).

One of the reasons for restricting ourselves to vectors of the form (6) shall become apparent next. Considerer the set of non-negative integers Z^+ and let the Cartesian product $Z^+ \times Z^+$ stand for vectors $\{|p+n-1,n>|\}$. In the set builder notation, the solution set

$$\{(m_1,m_2) / m_1 - m_2 = p-1, m_2 \geq m_1\}$$

ie the points $(p-1,0),(p,1),(p+1,2),\ldots,(p+n-1,n),\ldots$, are the basis vectors (6) of an order p parabose subspace. We may visualise these subspaces as straight-lines $m_2 = m_1 - (p-1)$ strolled up and down by the operators B^+, B respectively. Note that the parabose subspaces are constrained to lie in the first quadrant of the $B|0>=0$ plane because B|0=0, *ie* one cannot go below the point $(p-1,0)$ However, there is no upper limit for B^+ which moves upwards along the line $m_2 = m_1 - (p-1)$.

Conclusions

We have presented a bosonisation of the 1-D parabose oscillator where the state of n parabosons of order p is described by the Bose state $|p+n-1,n>$; the latter admits a geometrical description as a straight-line in the plane. The boson realization of the paraboson algebra was achived by expressing the paraboson operators B and B^+ as the product of two Bose operators with variable coefficients g given by (10), which may be interpreted as a coupling constant.

Being the oscillators of parabose and parafermi sort of complementary entities, the question arises of whether a Bose description of parafermions can also be viewed as points on a straight line. The answer is positive (11). We hope to report on this matter in not too distant a future.

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