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# Magnetic model of light in moving media and effects of the Aharonov-Bohm type*

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## Abstract

The magnetic model of light in moving media is considered here in relation to the electromagnetic interaction momentum that is linked to the Fresnel-Fizeau term. Applications of the magnetic model extend from gravitation to water waves. We show that the validity of the model might require that photons in moving media are not affected by the flow or that particles in the Aharonov-Bohm effect are dragged by the electromagnetic interaction. PACS: 03.30.±p, 03.65.Ta.

**Key words:** Electromagnetic momentum; light in moving media; quantum effects.

# Modelo magnético de la luz en medios en movimiento y efectos del tipo Aharonov-Bohm

## Resumen

El modelo magnético de la luz en medios en movimiento se considera en relación con el momento electromagnético de interacción que está vinculado al término de Fresnel-Fizeau. Aplicaciones del modelo magnético se extienden desde la gravitación a las ondas de agua. Se muestra que la validez del modelo requiere que los fotones en medios en movimiento no sean afectados por el flujo, o que las partículas en el efecto Aharonov-Bohm sean arrastradas por la interacción electromagnéticas. PACS: 03,30±p, 03,65. Ta.

**Palabras clave:** Efectos cuánticos; luz en medios de movimientos; momento electromagnético.

## Introduction

The formal analogy between the wave Equation for light in moving media and that for charged matter waves has been pointed out by Hannay (1) and later addressed by Cook, Feran, and Milonni (2-5). These authors have suggested that light propagation at a fluid vortex is analogous to the Aharonov-Bohm (AB) effect, where charged matter waves (electrons) encircle a localized magnetic flux (6). In quantum effects of the AB type (6-14) matter waves undergo an electromagnetic (em) interaction as if the waves were propagating in a flow of em origin

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that acts as a moving medium (14) and modifies the wave velocity. A magnetic model of light propagation in moving media and its relativistic theory has been elaborated by Leonhardt and Piwnicki (LP) (15,16). Besides the connection between the Fizeau effect and the AB effect in a real material medium, it has been shown that a non-uniformly moving medium appears to light as an effective gravitational field for which the curvature scalar is nonzero (15,16) and an analogue of the Fizeau effect for massive and massless particles in an effective optical medium has been derived from the static, spherically symmetric gravitational field (17-19). The magnetic model of light waves in moving media has been extended to water waves by Berry *et al.* (2-5), while acoustical analogs have been observed in moving classical media and should be visible in superfluids.

The existing analogy between the wave Equation s for matter and light waves can either be a formal similitude, devoid of a deeper physical meaning, or else a physically meaningful analogy that involves an interaction of the same physical nature. We show that the em interactions involved do indeed possess the same physical origin that is a common feature in the two Equation s (20). The Equation  for matter waves and light waves and its solution may be written (in units of $\hbar$= 1) as

$$(-i\nabla - Q)^2 \psi = p^2 \psi, \psi = e^{i\phi}\psi_0 = e^{i\int Q\cdot dx}\psi_0 \qquad [1]$$

where $\Psi_0$ solves (21) Equation [1] with **Q**= 0. Although the phase $\phi$ can be removed by a phase transformation, the phase shift, or phase shift variation, is an observable quantity that is phase (or gauge) invariant. With **Q**= [*e*/*c*]**A** where **A** is the vector potential, Equation [1] is the Schrödinger Equation for the magnetic Aharonov-Bohm effect, i.e., for a charged matter wave in a magnetic field where the flow u acts as a vector potential. **Q** has been expressed as (6-14)

$$Q = \pm P_e = \pm \frac{1}{4\pi c}\int (E \times B)\, d^3 x' \qquad [2]$$

where **E** is the electric field, **B** the magnetic field, and the quantity  is the linear momentum of the interaction em fields. The AB term **Q**= [e/c]**A** is obtained by taking **E** in Equation [2] to be the electric field of the charge and **B** to be the magnetic field of the solenoid. A general proof that this result holds in the *natural* Coulomb gauge, is given by Boyer, Zhu and Henneberger, and Spavieri (22-24). For the magnetic model of light propagation in moving media (2-16) in agreement with Fresnel's predictions and Fizeau's experiment, one sets in Equation [1] **p**→**k**= *nw*/*c* and we have the Fresnel-Fizeau term

$$Q = \frac{(n^2 - 1)}{c^2}\omega u \qquad [3]$$

where *n* is the index of refraction, **k** the wave vector, and X the angular frequency. The Fresnel-Fizeau term leads to a dragged light wave speed that agrees with the predictions of special relativity and reads $v_\phi = \frac{c}{n} + \left(1 - \frac{1}{n^2}\right)u$. From Equation [1], LP (15, 16) obtain the Hamiltonian of light rays (*H* is equal to the frequency X) and Hamilton's Equation s,

$$H = \omega = \frac{c}{n}k + (1 - \frac{1}{n^2})\, u \,.k \qquad [4]$$

$$\frac{dx}{dt} = \frac{\partial H}{\partial k}, \quad \frac{dk}{dt} = -\frac{\partial H}{\partial x}. \qquad [5]$$

The Equation s of motion for the moment a **p** and **k** are respectively

$$\frac{dp}{dt} = \mathrm{v} \times (\nabla \times \mathrm{Q}), \frac{dk}{dt} = -\left(1 - \frac{1}{n^2}\right)\nabla(u\,.k). \qquad [6]$$

The Equation  for **p** is analogous to the one derived by LP for the rescaled ray velocity **w**= *k***v** which reads

$$\frac{d\mathrm{w}}{dt} = -\left(1 - \frac{1}{n^2}\right)\mathrm{w} \times (\nabla \times \mathrm{u}). \qquad [7]$$

The analogy implied by Equation [1] is corroborated by the fact that the Equation of motion for the rescaled momentum **w** of light (photon) is the same as the Equation of motion for the momentum **p** of particles. Now we wish to establish the relation between the interaction momentum **Q** for light and the linear momentum of the em fields $\mathbf{P}_e$. In general, with $T_{ik}^M$ the Maxwell stress-tensor, the covariant description of the em momentum leads to the four-vector em momentum $P_e^\alpha$ expressed as [*c*= *1*].

$$P_e^i = \gamma \int (g + T_{ik}^M \beta^i) dV, \quad P_e^0 = \gamma \int (u_{em} - \mathrm{v} \,.\, g) dV \qquad [8]$$

where $\beta = v/c$, and the em energy and momentum are evaluated in a special frame $K^{(0)}$ moving with velocity **v** with respect to the laboratory frame. The standard classical-quantum correspondence $(\hbar = 1) \int u_{em} d^3x' = \frac{1}{4\pi} \int \varepsilon (E_0^2) d^3x' \to n^2 \omega_0$, $c^{-1} \int g d^3x' \frac{c^{-1}}{4\pi} \sqrt{\varepsilon}\, e_o \int (E_o^2) d^3x' \to k_o$ holds for the energy $\varepsilon \omega_o$ and the momentum $\mathbf{k}_0$. The natural choice for $K^{(0)}$ is the rest frame in which the momentum $\int (g) d^3 \sigma$ vanishes, i.e., the frame commoving with the light ray, as if the em mass $m_e$ of the fields were non-vanishing. We find that the em momentum $\mathbf{P}_e$ in the laboratory frame, and $\mathbf{P}_{oe}$ in the frame commoving with the fluid, are $\boldsymbol{P}_e = \gamma_v m_e v = \frac{\gamma_v}{\gamma_{c/n}} n^2 \frac{\omega_o}{c^2} \mathrm{v}$, $\boldsymbol{P}_{oe} = \gamma_{c/n} m_e \frac{c}{n} = n^2 \frac{\omega_o}{c^2} \frac{c}{n}$ With the help of $\gamma_v = \gamma_\mu \gamma_{c/n} (1 + \boldsymbol{u} \cdot \boldsymbol{c} / nc)$ for transformation of relativistic velocities we obtain

$$P_e - P_{oe} = \frac{\omega}{c^2} n^2 \mathrm{u} \qquad [9]$$

in the first order in $u/c$. We see from [9] that the variation $\mathbf{P}_e - \mathbf{P}_{oe}$ provides only the leading term of the Fresnel-Fizeau momentum. The fact is that $\mathbf{P}_e$ is the total em momentum while, according go (20), **Q** is linked to the em momentum of polarization $\boldsymbol{P}_i = [(n^2 - 1)/n^2] \boldsymbol{P}_e$ i.e., the fraction of the total em momentum due to interaction (polarization) which is proportional to $n^2 - 1$. From [9] we obtain the interaction polarization em momenta

$$P_i = (n^2 - 1) \frac{\gamma_v}{\gamma_{c/n}} \frac{\omega_o}{c^2} \mathrm{v}, \quad P_{oi} = (n^2 - 1) \frac{\omega_o}{c^2} \frac{c}{n}. \qquad [10]$$

$$P_i(u) - P_{oi} = (n^2 - 1) \frac{\omega_o}{c^2} \left( \frac{\gamma_v}{\gamma_{c/n}} \mathrm{v} - \frac{c}{n} \right) \simeq \frac{\omega}{\chi^2} (\nu^2 - 1) v. \; [11]$$

The term at the rhs of Equation [11] represents the exact relativistic variation $\boldsymbol{P}_i(\boldsymbol{u}) - \boldsymbol{P}_{oi}$ of the interaction polarization em momentum. The last term of Equation [11], the variation $\boldsymbol{P}_i(\boldsymbol{u}) - \boldsymbol{P}_{oi}$ in first order in $u/c$, is the Fresnel-Fizeau momentum **Q** of Equation [3], i.e., is the dragged interaction em momentum. We introduce here the rescaled momentum

$$\frac{n}{c} \mathrm{w} \equiv \frac{n}{c} k \mathrm{v} = k + \frac{\omega}{c^2} (n^2 - 1)\, u = K + Q, \qquad [12]$$

which can be interpreted in terms of the em momentum due to polarization. For the rescaled momentum $k[n/c]\mathbf{v}$, the relation $d\mathbf{u}/dt = (\mathrm{v} \cdot \nabla) \boldsymbol{u}$ and Hamilton's Equation s [5] lead to the Lorentz-type Equation of motion

$$\frac{d}{dt} \left( \frac{n}{c} \mathrm{w} \right) = \frac{d}{dt} \left( \frac{nv}{c} k \right) = (\nabla \times Q) \times \mathrm{v} \qquad [13]$$

where $\mathbf{Q} \to \mathbf{A}$ plays the role of a magnetic vector potential. We are able now to interpret the rescaled quantity $\mathbf{w}n/c$ appearing in Equation [12], Indeed with $\boldsymbol{P}_{oi} = k\boldsymbol{e}$ Equation [11] becomes Equation [12], implying that the rescaled quantity $\mathbf{w}n/c = k\mathbf{v}n/c$ corresponds to the interaction polarization em momentum $\mathbf{P}_i$. Let us now consider in detail the physical consequences of the magnetic model.

a) Equation [1] suggests that the wave vector and the speed of the wave are modified to **k** and $v_\phi$ by the flow **u.** The speed $v_\phi$ agrees with the predictions of special relativity and with the experimental observations of Fizeau. The fundamental point here is to establish if the additional momentum $\boldsymbol{Q} = (n^2 - 1)w\boldsymbol{u} / c^2$ that adds to $\mathbf{k}_0$ is carried by moving medium or by the light particle (photon). In effects of the AB type the additional momentum **Q** is carried by the medium, as the particle momentum **p** is not modified by the em flow. If the additional momentum $(n^2 - 1)w\boldsymbol{u} / c^2$ is localized and carried by the photon, the resulting canonical momentum **k** and the speed $v_\phi$ physically represent the momentum and speed of the light wave and particle (photon) dragged by the moving medium. Consider the propagation of light in a moving medium with characteristics analogous to those of the AB effect. In this case the velocity of the flow is such that $\boldsymbol{u}(x) \propto \boldsymbol{Q}(x) = (e / c)\boldsymbol{A}(x)$, where **A**(x) is a function that mimics the vector potential due to a solenoid. In the AB effect, charged particles coming with velocity **v** from far away and passing near the solenoid obey the Equation of motion [6], and their momentum **p** is not modified because, for this field-free effect, we have $\text{v} \times (\nabla \times \boldsymbol{A}) = 0$. However, if the momentum **k** represents the momentum of the light particles, a photon propagating through the flow $\boldsymbol{u}(x) \propto \boldsymbol{A}(x)$ modifies its momentum and velocity as implied by the Equation of motion [6] for **k** where $\nabla(\boldsymbol{u} \cdot \boldsymbol{k}) \propto \nabla(\boldsymbol{A}(x) \cdot \boldsymbol{k})$ does not vanish. Thus, there would be forces acting locally on the photon and the magnetic model and its analogy with the force- and field-free AB effect would break down.

b) If the magnetic model of light propagation in moving media holds in general, the additional momentum $(n^2 - 1)w\boldsymbol{u} / c^2$ is carried by the medium (as in AB effects) and not by the light particles. Since $\mathbf{k}_0$ does not change, the relationship $w = kc / n$ still holds and the group velocity remains $dw/dk = c/n$. However, the relativistic transformation of momentum **k** does not hold (it holds only if the total momentum $k_o + (n^2 - 1)w\boldsymbol{u} / c^2$ (momentum of the particle + interaction momentum with the medium) is considered).

In conclusion, in close analogy with matter waves of the effects of the AB type, the interaction momentum **Q** for light in moving media has been related to the linear momentum of the em fields $\mathbf{P}_e$. The value of **Q** calculated for light waves yields exactly the Fresnel-Fizeau momentum foreseen by special relativity. The momentum **k** of Equation [6] and $\mathbf{P}_i$ of Equation [13] obey analogous Equation s of motion, and both matter and light waves obey the same wave Equation [1] [if $\hbar$= 1).

However, the analogy seems to hold for the wave velocities but it is not clear if it holds also for the particle velocities. We stress that the traditional Fizeau-type of experiments are based on interferometric measurements that search for phase shifts of phase velocity variations of the waves, but not the velocity of the particles as such (similarly, in effects of the AB type). There have been no dedicated experimental tests of the speed of photons in moving media, or of particles in effects of the AB type. To account for all alternatives, one could also consider the possibility that in effects of the AB type there may be forces acting on the massive particles. We do not favor this possibility, but it has been considered in the literature within the context of accepted classical electrodynamics (25-30) and within the so called stochastic electrodynamics (SED) theory (31, 32).

Thus there are analogies but also some discrepancies between matter and light waves (or electrons and photons). Because of all these contrasting theoretical views, it seems worthwhile to investigate the feasibility of experimentally testing the magnetic model of light (33). The various theoretical scenarios discussed above could be investigated, and either confirmed or ruled out, by new non-interferometric tests of photon and particle speeds under appropriate conditions (14).

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