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Compact stars cooling and secular instability*

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Abstract

Compact stars cooling theory is revised thought secular stability criteria, this is when the relaxation times are not negligible. Using the Cattaneo Law for the heat flux, it is shown changes in the energy transport equation and insinuates quasiperiodic pulses in the luminosity. Applications in rapid variations in single white-dwarf oscillators and quasi periodic luminosity pulses of Neutron Stars (NS) are suggested. Also we discussed the Secular Stability in milliseconds emissions Pulsar.

Key words: Instabilities - cooling time; convection; pulsar: interior; stars: compact.

Enfriamiento de estrellas compactas e inestabilidad secular

Resumen

La teoría de enfriamiento de estrellas compactas es revisada a través del criterio de estabilidad secular, cuando el tiempo de relajación no es despreciable. Usando la Ley de Cattaneo para el flujo de calor, se muestran los cambios en la ecuación de transporte de energía y se insinúan emisiones cuasi-periódicas en la luminosidad. Se sugieren aplicaciones del formalismo en el estudio de las variaciones de Estrellas Enanas Blancas aisladas y en pulsaciones de estrellas de Neutrones. También se discute la Inestabilidad Secular en la emisión de pulsares de milisegundos.

Palabras clave: Estrellas: compactas; interior- tiempo de enfriamiento; convección; pulsares: interior.

Introduction

The thermal properties are those responsible for the radiation and further evolution of White Dwarfs (WD) and Neutron Stars (NS). However in standards calculation of the cooling time (1, 2) the possibility of the heat propagation by waves is obviated. This simplification will be spurious in the degenerated material because, the relaxation time is not neglecting. Also inside NS layers, constituted for superfluid helium II, the relaxation time as high as hundred seconds and the luminosity will oscillate (3). In this

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article, it is described how changes the cooling time and the luminosity in WD and NS stars if heat waves are taken into account. To make this, the Maxwell-Fourier is replaced by the Cattaneo causal law in the energy transport equation (section 2), the cooling theory is revised in section 3 and 4 for NS and WD respectively, and the section 4 we discussed the secular stability in the pulsar interior. The conclusions are show in the last section.

2. Cooling of Neutron Stars

If the energy in the stars is transported through the star layer by radiation, conduction and/or convection, the temperature gradient is given, in the stellar interior, in terms of local values of opacity κ density ρ and energy flux F by the relation (2-4):

$$\frac{dT}{dr} = \frac{3\kappa\rho}{4acT^3}F \qquad [1]$$

This relation is based in Fourier-Maxwell law, according to which perturbations propagate with infinite speed. A heat flux equation leading to a hyperbolic equation (telegraph equation) is the Cattaneo law (5):

$$\vec{F}(\vec{x},t) = -\frac{k}{\tau}\int_{-\infty}^{t}\exp[-(t-t')/\tau].\vec{\nabla}T(\vec{x},t')dt' \qquad [2]$$

For the study of the cooling of the NS we shall closely follow the approach given by ref. [4], keeping in as much as possible their notation and changing the energy transport equation. The total luminosity L= $-4\pi R^2$ F, is

$$L = -\frac{4\pi Rk}{\tau}\int_{-\infty}^{t}DT.\exp[-(t-t')/\tau]dt' \qquad [3]$$

where we have used that $\nabla T \approx DT/R$ ($T_{Central}$-$T_{Surroundings}$)/R. By the other hand the rate by which the thermal energy changes is given by: $L = C_V\frac{dDT}{dt}$ and the equation [3], we obtains:

$$\frac{sDT}{dt} = \frac{1}{\tau\xi}\left[\int_0^t DT(t')\exp[-(t-t')/\tau]dt' + \chi.\exp(-t/\tau)\right] \qquad [4]$$

with

$$\chi \equiv \int_{-\infty}^{0}\Delta T(t')\exp(t'/\tau)dt' \approx \tau.DT(0);\ \xi \equiv \frac{C_V}{4\pi Rk} \qquad [5]$$

Taking Laplace Transform of both sides and some elementary algebra, then:

$$DT0t = DT(0).e^X\left\{\cos(wx) - \frac{sen(wx)}{w}\left[1+\frac{2\chi}{4\pi Rk}\right]\right\} \qquad [6]$$

with $X \equiv t/2\tau$; $W \equiv \sqrt{\frac{4\pi}{\xi}-1}$. Then feeding back [6] into [3] we obtain:

$$L = [4\pi RkDT(0)\exp(-t/\xi)].f(x,w) \equiv L_o.f(x,w) \qquad [7]$$

where

$$f(x,w) = \frac{1}{\omega^2+1}.\left[(5+\omega^2).\cos(\omega x) - \frac{(3-\omega^2)}{\omega}.\sin(\omega x)\right] \exp\left[\frac{x}{2}.(\omega^2-1)\right] \qquad [8]$$

The last term in [7] defines the standard luminosity in the Maxwell Fourier (L_O) regimen (when the relaxation time is negligible). The Equation [7]) connects the standard luminosity and the "true" luminosity before thermal relaxation (in presence of heat waves). Let us consider a normal NS witch temperature T_9 (units of thousand million degree) and total thermal energy (4) U= A T_9 . Now, inserting the appropriate luminosity into cooling equation, we obtain $\int_0^{fi} dU = -\int_0^{fi} L.dt$ But the luminosity in [7] is time dependent and the new cooling time is defined as e-folding time. The cooling equation is now

$$\tau_V = \frac{A}{\langle L \rangle}\left[T^2(fi) - DT^2(0)\right] \qquad [9]$$

where $\langle L \rangle$ is the average in the interval $(0,\xi)$. This interval has been defined by e-folding time and it is not restrictive in this discussion. We can see easily that the value average of that oscillatory functions (L) is very different of the corresponding to a monotonous luminosity function(L_O).Combining [6] and [9] it is clear that the cooling time could be bigger or smaller than the usual cooling time of NS depending on the values assumed for relaxation time.

3. The Cooling Time of White Dwarfs

The influence and importance of the convection and mixing length theory (MLT) in the study and calculations of the atmosphere model for white dwarf is highly report (Ref. (6) and references therein).

If the WD core is degenerated (or with layers superfluid helium) then the relaxation time couldn't be this negligible. In this case the WD luminosity admits quasi periodic variation while the times are less than relaxation time. Using the convection theory for the energy transport in the regime of Cattaneo it finds that the luminosity has the form of equation [7] (7). We assume the Cattaneo law for the temperature gradient similarly to [1] in the condition of hydrostatic equilibrium, may also be written as:

$$\frac{dT}{dP} = \frac{3\kappa}{64\pi\sigma}\cdot\frac{1}{GMT^3}\cdot\left(\tau\frac{\partial L}{\partial t} + L\right) \qquad [10]$$

Notice that if $\tau \approx 0$ the relation [10] is the "classical" equation for transport of energy in the stars interior. The presence of a sub surface convection zone can highly affect the WD rate cooling and its age, as to be seen now. According to. [7] and [10] the energy transport equation is given as:

$$T^{3-j}\cdot P^{-i}\frac{\partial T}{\partial P} = \frac{3\kappa_0}{64\pi\sigma}\left(L^{(d)}/GM\right)\cdot$$
$$\left[\left(1-\frac{\tau}{\tau_d}\right)\cdot f(x,w) + \tau\frac{\partial f(x,w)}{\partial t}\right] \qquad [11]$$

where we used the Kramers opacity law. Now, we simplify matters by assuming a discontinues transition from degeneracy to non degeneracy to certain point (subscript 0). We have (8) that:

$$T_0 \cong \vartheta^{2/7}\left(L^{(d)}/L_\Theta\right)^{2/5}\left(M/M_\Theta\right)^{-\frac{2}{5}} g^{2/7}(x,w) \qquad [12]$$

where T_0 is written in term of the Sun mass (M_Θ) and Sun luminosity (L_Θ) and the energy equation allow write the luminosity as:

$$-L \approx c_v\frac{\partial T_0}{\partial\tau}M \qquad [13]$$

Feeding back [11] and [7] into [13], after some integration from 0 to τ_V (life time or the cooling time) then:

$$\int_0^{\tau_V}\left(1+\frac{\tau}{\tau_d}+\frac{\tau}{f(x,w)}\cdot\frac{\partial f(x,w)}{\partial x}\right)^{-1}$$
$$dt = \frac{2}{5}\left(\frac{M_\Theta}{L_\Theta}\right)c_v\vartheta T^{-\frac{5}{2}} \qquad [14]$$

Obviously when $\tau \approx 0$ (without heat waves) the cooling time is the very well know, WD cooling time ($\tau_V^{(d)}$), in the classical literature about subject. In general, if the heat waves existent then cooling time increase. The general solution of Equation [14] is:

$$A_1\tau_v + A_2\cdot\left|\ln\left[\frac{q_1\sin(\omega x)+p_1\cos(\omega x)}{p_2\sin(\omega x)+q_2\cos(\omega x)}\right]\right|_{x=0}^{x=2\tau_v/r} = \tau_V^{(d)}$$

[15]

where q_1, q_2, p_1, p_2, A_1 y A_2 are coefficient in function of τ and τ_d. It is easy to look that the non-lineal term in left member of the Equation [15] is restrained.

4. Secular Instability

Double diffusive instabilities arise when the transport of two different properties compete against each other to dominate the stability of a fluid, in stars, heat and composition are the quantities transported by mixing and convection (1, 2). In a fluid distribution layers of higher molecular weight above a colder region of lower molecular weight may be dynamically stable if the specific weight of the former is reduced, because of its higher temperature, below that of the underlying layer. If a blob of the upper layer is pushed downward, buoyancy will push it back. However on the time scale by which the blob loses it's excess temperature (DT), the buoyancy decreases and the blob sinks. This secular instability (Thermohaline Instability) is controlled by heat leakage of the blob (2). Under these conditions the blob speed is:

$$V_d = -\frac{H_p}{(\nabla_{ad} - \nabla)}\frac{DT}{\tau_d T} \qquad [16]$$

Where ∇_{ad} y ∇ are the logarithmic variation of the temperature respect to pressure, under constant entropy constant and head constant respectively, Hp is the scale height of pressure and τ_d is the thermal adjustment time (See ref. (2) for detail).

In NS with upper layers of helium superfluid, the radial convective motions could be present (secular instability). The motions of the convective globules with different molecular weight, would cause the variations in the radial speed of the globule given by (9):

$$V = V_d\left(\tau_d/\tau\right)\exp\left[2x\tau/\tau_d - 1\right]\left\{\cos(\omega x)\left(2 + \frac{2\chi}{DT(0)\tau_d}\right) + \sin(\omega x)\left[\omega - \frac{1}{\omega}\left(1 + \frac{2\chi}{DT(0)\tau_d}\right)\right]\right\} \qquad [17]$$

This show that the convective blob does not always move down the temperature gradient before relaxation, but in the opposite direction during short periods of time. The blobs oscillating in compact radio pulsar sources will induce oscillations in the rotation rate and thereby micro fluctuations in the emission rate. This process could be good model by quasiperiodic microstructure observed in five radiopulsars, reported by Cordes et al 1990 (10). They find a periodic microstructure with quasiperiods ranging from 0.5 to 5 ms, the same as the order of magnitude expected for the relaxation time in NS.

Of agreement with the magnetic dipole model the pulsar radiate energy at rate (11):

$$\frac{dE_{rot}}{dt} = -\left(\frac{B_p^2 R^6 sen^2\alpha}{6}\right)\Omega^4 \equiv -\beta.\Omega^4 \qquad [18]$$

where Ω is the velocity angular, and α is the inclinations of the magnetic field (B) with the rotation directions. The variation of energy carried away by radiation originates from the rotational kinetic energy is given by:

$$\frac{dE_{rot}}{dt} = I\Omega\frac{d\Omega}{dt} + \frac{\Omega^2}{2}\frac{dI}{dt} \qquad [19]$$

This Equation can be integrated to obtain:

$$t \approx \frac{T}{2}\ln\left|\left(\frac{\Omega_0}{\Omega_\iota}\right)^2 + \frac{2\beta}{\dot{I}}\right| \qquad [20]$$

where

$$\dot{I} \equiv \frac{dI}{dt} \approx 2\int \vec{r}\,.\,\vec{v}\,dm \qquad [21]$$

It is the variation temporary of the moment of inertia (I), due to the movement of a globule of mass dm, with speed v, located in the position r. Into NS degenerate material, the convective globules presents radial movements quasi periodic (with ranging from milliseconds to seconds) in scale of the relaxation time, this variations caused the fluctuations in the Inertia moment, and the quasi periodic variations in the flux of the energy.

5. Conclusions

The Equation [7] show that the luminosity is dependently of the previous history of the temperature gradient and the envelope composition. This result open new possibilities not foreseen in the recent studies of fast cooling of Neutron Stars (12). Also the Equation [9] show that the cooling time could be very different than the usual NS cooling time, depending on the values assumed for τ, in the special case $\tau \approx \xi$ we obtain that the NS cooling time diminish.

In the coolest WD, the Equation [14] suggests that the relaxation time increases and the propagation of heat waves must be considered in the later phases. The relaxation time for WD and quasi periodic variation of luminosity could be a model of the single white dwarf oscillation (ZZ ceti stars) because the typical period is a few hundred seconds (13). If $\tau \approx \tau_d$ (critical case) then $\tau_V \approx 2\tau_V^{(d)}$: the age of WD will be two times greater when the causal propagation of heat s considerate. Also the WD cooling time (coolest objects) is very sensitive to luminosity and this can charge the results for the oldest stellar ages and the galactic disk.

The relations [20] and [21] alleged that quasi periodic oscillations observing in QPO could be "naturally" modeled for instability criteria whereas the blobs (and hot spots) generate for convection in stars atmosphere before thermal relaxation. Also, this quasi-periodic oscillation in the millisecond range is the order to relaxation time for NS (9, 11). As the speed of the globule agreement oscillates cuasiperiodically with the equation [17], the inertial moment variation (dI/dt) will also be an oscillatory function with the same period. It is clear that the oscillation time t, given in the equation [20], will also present fluctuations of order τ.

Also, in the specific case of a compact X-ray source (10, 14) in a binary system, accreted matter may form an outer layer, creating conditions for secular instability. The blobs oscillating in compact X-ray sources will induce oscillations in the rotation rate and thereby oscillations in the emission rate. In a future, the more specific model could be calculations based in the preliminary results above description.

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