CIENCIA 3(2), 109-116, 1995
Maracaibo, Venezuela

Transition regime effective viscosity of a gas confined in a cylindrical geometry

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Recibido: 13-03-95 Aceptado: 21-06-95

Abstract

The effective viscosity of a gas in the transition regime confined to a cylindrical geometry is obtained using the path integral method. In the transition regime a pattern of rapid variation of the effective viscosity with the density in equally spaced narrow regions of pressure is found. Those regions are quite unaltered when boundary values are either constant, or variable. For the last case. linear. oscillatory, and random dependences with density have been considered. A discussion is given relating these results to those previously obtained for a planar geometry of two parallel infinite plates.

Key words: Gas: transition regime; viscosity.

Viscosidad efectiva de un gas en régimen de transición confinado en una geometría cilíndrica

Resumen

La viscosidad efectiva de un gas en régimen de transición confinado en una geometria cilíndrica es obtenida haciendo uso del método de la integral de camino. Regiones de rápida variación de la viscosidad efectiva aparecen en el régimen de transición equiespaciadas en presión. Dichas regiones corresponden a pequeños intervalos en presión y varían poco con modificaciones de las condiciones de contorno. El efecto sobre la viscosidad efectiva de variaciones lineales, oscilantes y aleatorias de las condiciones de contorno con la presión ha sido examinado. Se discute finalmente el presente modelo comparándola con uno previamente desarrollada para el caso de una geometria plana de *dos* planas paralelos infinitos.

Palabras claves: Gas; régimen de transición; viscosidad.

1 Introduction

The transition regime of gases is that range in pressure where the Knudsen number: $K_n \equiv \ell/L$, is near unity: ℓ and L being the mean free path and a characteristic macroscopic dimension of the system, respectively. As is well known, the viscosity of gases in the continuous regime ($K_n << 1$) is independent of Kn (1) provided that the density is not too high (2.3) .In that regime the viscosity is an intrinsic property of the gas, while in the transition ($K_n \approx 1$) and Knudsen ($K_n >> 1$) regimes the viscosity

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depends on the flow geometry. It is for this reason that in the last two cases the concept of effective viscosity defined as the ratio of the actual stress to the continuous regime stress is used (4,5).

First order gradient expansions of the stress do not explain flows in the transition regime (3,6-8) since higher order equations in the Chapman-Enskog expansion appear to be necessary (2,3). Yet, the boundary conditions for these high order equations are unknown, although several approximate treatments of this problem have been developed (2,3,8,9). On the other hand, although boundary conditions may be expected to depend on the Knudsen number K_n, there are reasons to believe that this dependence may be weak in the transition and Knudsen regimes (see below). Moreover, we show here that the expression obtained for the effective viscosity, assuming boundary values independent of K_n, presents a period of oscillation which is quite insensitive to different types of variation of the boundary values with K_n. This allows a simplification of the study of the transition regime effective viscosity of a gas in a cylindrical torsion pendulum viscometer.

Following a simple path integral method previously described (4), one obtains a Burnett-type expression (8,9) for the stress tensor component

$$P_{\rho\phi} = -\eta \frac{\partial \upsilon_\phi}{\partial \rho} - \gamma \frac{\partial^3 \upsilon_\phi}{\partial \rho^3}, \qquad [1]$$

where higher order terms $A_n(\partial^n \upsilon_\phi / \partial \rho^n)$, with $n = 5,7,\ldots$, have been neglected, ρ and ϕ being the radial and angular cylindrical coordinates, $\upsilon_\phi(\rho)$ the fluid velocity, (Figure 1) and

$$\eta = n\,KT\tau\,,\ \gamma = 3n\,m\tau^3\left(\frac{kT}{m}\right)^2, \qquad [2]$$

where n is the number of molecules per unit volume and τ is the average-time between collisions. In the above equation η is the usual coefficient of viscosity, and γ the coefficient of the first corrective term appearing when the mean free path is no longer neglected in front of a characteristic macroscopic dimension of the system (4,10). As can be seen, the boundary conditions for Equation [1] are naturally the values of υ_ϕ and $\partial\upsilon_\phi/\partial\rho$ on the fixed and moving surfaces of our cylindrical viscometer. Let us denote these boundary values as:

$$\upsilon_1 \equiv \upsilon_\phi(\rho = \mathbb{R}_1)\,,\ \upsilon_2 \equiv \upsilon_\phi(\rho = \mathbb{R}_2) \qquad [3]$$

and

$$a_1 \equiv \frac{\partial \upsilon_\phi}{\partial \rho}\Big|_{\rho=\mathbb{R}_1},\ a_2 \equiv \frac{\partial \upsilon_\phi}{\partial \rho}\Big|_{\rho=\mathbb{R}_2}. \qquad [4]$$

where $\rho = R_1$ and $\rho = R_2$ correspond to the fixed and the moving surfaces of the viscometer, respectively. To get the friction on the torsion pendulum, Equation [1] must be solved twice, first for the inner fixed surface and the interior face on the moving surface, and secondly, for the exterior face of the moving surface and the exterior fixed surface: being the total friction equal to the sum of these two contributions.

In Sec. II we will consider the problem of the local equilibrium function for a gas that rotates with a given angular velocity. In Sec. III the viscosity of a dilute gas in a centrifuge field of force is calculated by the path integral method up to third order in the average-time between collisions τ. A pattern of rapid variation of the effective viscosity within equally spaced narrow regions of K_n is obtained, in qualitative agreement with the experimental results already reported (5, 10, 11). Finally, in Sec. IV some concluding remarks are presented.

II. Local equilibrium function for a gas that rotates with a given angular velocity

Let us consider a dilute gas, enclosed in a cylindrical recipient of radius R and height H, that rotates around the symmetry axis with constant angular velocity Ω. To describe this system we used two cylindrical coordinate systems: one fixed in the laboratory, and the other rotating with angular velocity Ω, but having the same origin and the same Z axis as the fixed system (Figure 1). If we denote $\vec{U}$, the thermal velocity of a molecule measured in the rotating coordinate system, and $\vec{V}$ the velocity of the same molecule measured in the fixed coordinate system, we can write

$$\vec{V} = \vec{U} + \vec{\Omega} \times \vec{r}, \qquad [5]$$

where $\vec{r}$ is the position vector of the molecule in the fixed coordinate system. The term $\vec{\Omega} \times \vec{r}$, in Equation [5] is the drift velocity of the fluid over the thermal velocity of the molecules. The corresponding energy in the rotating coordinate system is given by:

$$E = \vec{p}.\, \vec{U} - L\,, \qquad [6]$$

where L is the Lagrangian $mv^2/2$ and

$$\vec{p} = \frac{\partial L}{\partial \vec{U}} = m\vec{U} + m\vec{\Omega} \times \vec{r}.$$

From Equations [5] and [6], one obtains for the energy in the rotating system

$$E = \frac{mU^2}{2} - \frac{m}{2}\,|\vec{\Omega} \times \vec{r}\,|^2 \qquad [7]$$

Once we have the above expression, we can write the local equilibrium distribution function as:

$$f^{(0)} = C \exp\left(-\frac{E}{kT}\right), \qquad [8]$$

where C is a normalization constant. Using the cylindrical radial variable $\rho = r \operatorname{sen}\theta = [x^2 + y^2]^{1/2}$, where θ is the angle between r and Ω, it can be shown from Equations [7] and [8], that the local distribution function can be written as

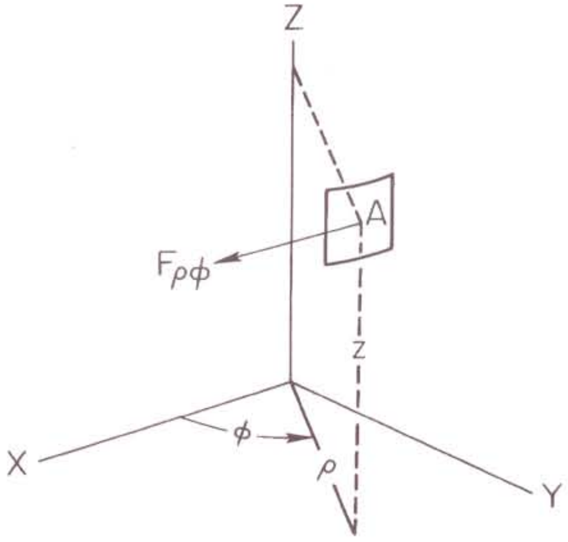

Figure 1. The force component $F_{\rho\phi} = AP_{\rho\phi}$, where $P_{\rho\phi}$ is the stress tensor component, on a section fluid of area A.

$$f^{(0)}(\rho, \vec{U}) = n(\rho)\left[\frac{m}{2\pi kT}\right]^{3/2} \exp\left[-\frac{mU^2}{2kT}\right], \qquad [9]$$

where n (ρ) is the local density:

$$n(\rho) = \frac{N \exp\left(\dfrac{m\Omega^2\rho^2}{2kT}\right)}{\dfrac{2\pi HkT}{m\Omega^2}\left[\exp\left(\dfrac{m\Omega^2 \mathbb{R}^2}{2kT}\right) - 1\right]} \qquad [10]$$

In order to get Equation [9] the constant term C appearing in Equation [8] should be determined by the condition that the total number of molecules N be equal to:

$$N = \int f^{(0)}\, d^3U d^3r\,, \qquad [11]$$

where $d^3 r = \rho d\rho d\phi dz$ and $d^3U = U_\rho\, dU_\rho\, d\beta\, dU_z$, β being in the velocity space, what ϕ is in the coordinate space. In the determination of C, straightforward calculations of integrals of the type $\int_a^\infty e^{-\alpha X^2} X^n dX$, where a is either 0 or $-\infty$, are necessary (4,12). From Equations [9] and [10] it can be observed, that as a consequence of the rotation, a dependence of the density with the cylindrical radial variable ρ appears. On the other hand, it can be verified from Equation [10] that $\lim_{\Omega\to 0} n(p) = N/V = const$, where V is the volume of the cylinder.

III. Viscosity of a dilute gas in a centrifuge field of force, calculated by the path integral method

According to the path integral method (12), the molecular distribution function can be expressed as

$$f(\rho,\vec{U},t) = \int_0^t f^{(0)}(\rho_0,\vec{U_0},t-t') \exp\left(-\int_{t-t'}^{t} \frac{ds}{\tau(s)}\right) \frac{dt'}{\tau(t-t')} \quad [12]$$

Succesive use of the relation $d/dX \int_{X_0}^{X} f(s)\, ds = f(x)$ and the technique of integrations by parts on Equation [12], results in the following expression:

$$f = f^{(0)} + \tau\,(t-t')\,\frac{df^{(0)}}{dt'}\Big|_{t'=0} +$$

$$\tau(t-t')\,\frac{d}{dt}\left(\tau(t-t')\,\frac{df^{(0)}}{dt'}\right)\Big|_{t'=0} +$$

$$\tau(t-t')\,\frac{d}{dt}\left[\tau(t-t')\,\frac{d}{dt}\left(\tau(t-t')\,\frac{df^{(0)}}{dt'}\right)\right]\Big|_{t'=0} +\ldots \quad [13]$$

where $f^{(0)}$ is given by Equation [9] $U = U_\rho, U_\phi, U_z)$ and $U_\rho = v_\rho$, $U_\phi = v_\phi(\rho) - \upsilon_\phi(\rho)$ $U_Z = V_Z$.

Following a similar procedure to that previously described (4), but starting from Equations [9], [13] and the expression for the tangential stress $P_{\rho\phi} = m\int d^3\, U f U_\rho\, U_\phi$, a Burnett-type expression is found:

$$P_{\rho\phi} = -\, nKT\tau\,\frac{\partial \upsilon_\phi}{\partial\rho} - 3\tau^3\,\frac{K^2T^2}{m}$$

$$\left[2\,\frac{\partial^2 n}{\partial\rho^2} - \frac{3}{n}\left(\frac{\partial n}{\partial\rho}\right)^2\right]\frac{\partial \upsilon_\phi}{\partial\rho} - 3n\tau^3\,\frac{K^2T^2}{m}\,\frac{\partial^3 \upsilon_\phi}{\partial\rho^3} \quad [14]$$

The second term in the r.h.s. of Equation [14] takes into account the correction to viscosity coming from the density variation with ρ due to the rotation of the gas. For the sake of simplicity we will consider in what follows that this correction can be neglected, since, for instance, for $\Omega = 0.571\ s^{-1}$, $R_{max} = 25.5$ cm, T = 293 K and $m = 6.64 \times 10^{-24}$ g, we have from Equation [10] that the ratio of the factors proportional to τ^3, which respectively multiply $\partial\upsilon_\phi/\partial\rho$ and $\partial^3\upsilon_\phi/\partial\rho^3$ in Eq. (14), yield

$$\frac{1}{n}\left[2\,\frac{\partial^2 n}{\partial\rho^2} - \frac{3}{n}\left(\frac{\partial n}{\partial\rho}\right)^2\right]\Big|_{\rho=R_{max}} \cong 10^{-11}\ cm^{-2} \quad [15]$$

Modifications of the values for Ω and R_{max} given above by an order of magnitude produce a modification of at most two order of magnitude of the ratio given in equation [15]. This means that the second term in the right hand side of Equation [14] is negligible with respect to the other term proportional to τ^3 appearing in this expression in a wide range of situations. Yet, a combination of large curvature and high angular velocity will render this term important, a situation that will be analized elsewhere. Taking $n = N/V \approx n\ (\rho)$, the remaining terms of Equation [14], lead us to Equation [1]:

$$P_{\rho\phi} = -\eta \frac{\partial \upsilon_\phi}{\partial \rho} - \gamma \frac{\partial^3 \upsilon_\phi}{\partial \rho^3}, \qquad [1]$$

where η *y* γ have the same form given in Equation [2].

The above equation for the stress in terms of the cylindrical variables ρ and ϕ is formally identical with an analogous equation in terms of Cartesian variables that we have already analyzed in previous papers for the vanishing temperature gradient case (4). Yet, the cylindrical geometry appears through the boundary conditions discussed in the introduction (Equations [3] and [4]), which we use together with

$$2\pi H \rho P_{\rho\phi} = const. \qquad [16]$$

to solve Equation [1]. The above condition corresponds to that case in which concentric layers with zero relative acceleration, exist.

Using the change of variable $y = \rho\, \partial\, \upsilon_\phi / \partial \rho$ one obtains from Equations [1] and [16]

$$\frac{d^2 y}{dx^2} - \frac{2}{x}\frac{dy}{dx} + \left(1 + \frac{2}{x^2}\right) y = \frac{a}{\omega^2} \qquad [17]$$

where $x = \omega\rho$, and we define the constants

$$a \equiv -\rho \frac{P_{\rho\phi}}{\gamma}, \; \omega^2 \equiv \frac{\eta}{\gamma} \qquad [18]$$

Finally it can be shown that Equation [17] is equivalent to

$$\frac{d^2 V}{dx^2} + V = \frac{k}{x}, \qquad [19]$$

with $y = xV(x)$ and $k = a/\omega^2$. This linear nonhomogeneous equation can be solved using the method of variation of parameters (13), which yields:

$$V(x) = (C_1 + kSI(\alpha) - kSI(x))\, cosx +$$
$$(C_2 - kCI(\beta) + kCI(x))\, sinx \qquad [20]$$

where SI (x) and CI (x) are the well known integral functions (14)

Using the same change of variables previously used in passing from Equation [1] to Equation [19], one obtains:

$$\upsilon_\phi(x) = (C_1 + kSI(\alpha))sinx + (k\, CI\, (\beta) - C_2)cosx -$$
$$kSI(x)sinx - kCI\,(x)cosx + k\, Lnx + C \qquad [21]$$

which together with the boundary conditions given in Equations [3] and [4], finally lead to the component of the stress tensor:

$$P_{R_2\phi} = -\eta_0 \left\{ \frac{1 - \dfrac{a_1 + a_2}{\omega \Delta \upsilon} \tan \dfrac{\omega \Delta R}{2}}{\left[g_2 - g_1 + \ln(R_2/R_1) - (f_1 + f_2) \tan\left(\dfrac{\omega \Delta R}{2}\right)\right] \dfrac{R_2}{\Delta R}} \right\} \frac{\Delta \upsilon}{\Delta R}, \qquad [22]$$

where $g_1 \equiv g(R_1)$, $g_2 \equiv g(R_2)$, $f_1 \equiv f\,(R_1)$, $f_2 \equiv f(R_2)$ and

$$g(R) = \frac{\pi}{2}\sin\omega R - CI(\omega R)\cos\omega R$$
$$- SI(\omega R)\sin\omega R \qquad [23]$$

and

$$f(R) = \frac{\pi}{2}\cos\omega R + CI(\omega R)\sin\omega R$$
$$- SI(\omega R)\cos\omega R \qquad [24]$$

It can be shown that the limit R_1, $R_2 \to \infty$ of $P_{\rho\phi}$ given in Equation [22] with $\Delta R = L = const.$ leads to the expression of the stress tensor corresponding to a planar geometry given in Equation [41] of Ref. 4. On the other hand, the high density limit of Equation [22] gives also the correct continuous regime expression.

The component of the stress tensor $P_{R_2\phi}$ given in Equation [22] presents avoidable indeterminations of the type ∞/∞ when

$$\frac{\omega \Delta R}{2} \to (2M + 1)\frac{\pi}{2} \qquad [25]$$

for M integer. In contrast with the planar case, $P_{R_2\phi}$ remains finite for Knudsen number values corresponding to Equation [25]. These values of K_n fall in the narrow regions of rapid variation of $P_{\rho\phi}/P_{\rho\phi}^{(0)}$ appearing in Figure 2. These narrow regions are separated in density by

$$\Delta n = \frac{1}{\left(\frac{4}{3\pi^3}\right)^{1/2} \Delta R\sigma} \qquad [26]$$

where we have used $\omega = (8/3\pi)^{1/2}\sqrt{2}\,\sigma n$ (see Equation 18 and Equation 25). The relation between the Knudsen number K_n and the density being given by $K_n = 1/[(16/3\pi)^{1/2}\sigma n\,\Delta R]$.

The boundary conditions enter in the expression for $P_{\rho\phi}$, Equation [22], through the quantity

$$B(K_n) = \frac{a_1 + a_2}{\left(\frac{\upsilon_2 - \upsilon_1}{\Delta R}\right)} \qquad [27]$$

In Figure 2 we have represented $P_{\rho\phi}/P_{\rho\phi}^{(0)}$ as a function of pressure for a gas having a cross-section $\sigma = 80 \times 10^{-16}$ cm^2, T = 293 K, R_1 = 2.5 cm and R_2 = 5.4 cm. This rather high value of σ which may correspond to a polyatomic molecule like ethane, has been taken in order to illustrate a case in which the effective viscosity oscillates with pressure several times in the range going from 0 to 40 µm Hg. In Figure 2 $K_n = 1$ corresponds to 0.92 µm Hg in the horizontal axis. Although K_n explicity appears in the expression of $P_{\rho\phi}/P_{\rho\phi}^{(0)}$ we cannot deduce how this ratio depends on K_n, since the expression of $a_1 + a_2$ as a function of K_n will require a microscopic characterization of the surfaces along with a detailed knowledge of their interaction with the molecules of gas. However, we may expect $|B(K_n)|$ to be less than 1 in the transition and Knudsen regimes, since by definition of the mean free path ℓ, the molecules either approaching or departing from a surface will move with uniform linear velocities in a region of width $\bar{\ell} \lesssim \ell$ measured from the wall (10). Then for $K_n \sim 1$, i.e. $\bar{\ell} \sim R$, where R is a macroscopic distance, we may expect that $a_1 \simeq 0$ and $a_2 \simeq 0$ and therefore $|a_1 + a_2| << |(\upsilon_2 - \upsilon_1)|/\Delta R$ for usual tangential velocities of the moving surfaces.

We have considered several types of variations of $B(K_n)$ in order to evaluate the effect of the boundary conditions on the dependence of $P_{\rho\phi}/P_{\rho\phi}^{(0)}$ on K_n. In the first row of Figure 2 we have illustrated the case corresponding to boundary conditions independent of K_n, taking B = 0.5 and B = - 0.5, respectively. As can be seen in the second and third rows the pattern of the variation of $P_{\rho\phi}/P_{\rho\phi}^{(0)}$ remains quite unaltered as we consider successively linear, oscillating and random (between 0 and 1) variations of B with K_n. This result is relevant to the question whether or not the variation of the boundary conditions with K_n giving rise to a dependence of B with K_n (see Equations [22] and [27]), wash out the regions of rapid variation of the effective viscosity with density. According to the above results (Figure 2) this seems not to be the case, which gives support to our determination of σ from the experimental results of gases in the transition regime evaluating the separation Δn between regions of rapid variation of $P_{\rho\phi}/P_{\rho\phi}^{(0)}$ and then using Equation [26] to calculate σ that we have reported in Refs. 5, 10 and 11.

Let us stress that this type of non-monotonic dependence of the effective viscosity with density for gases in the transition regime, (given in Equation [41] of a previous paper (4) for the planar case, and in Equation [22] of the present paper for the cylindrical case), corresponds to a fluid in a

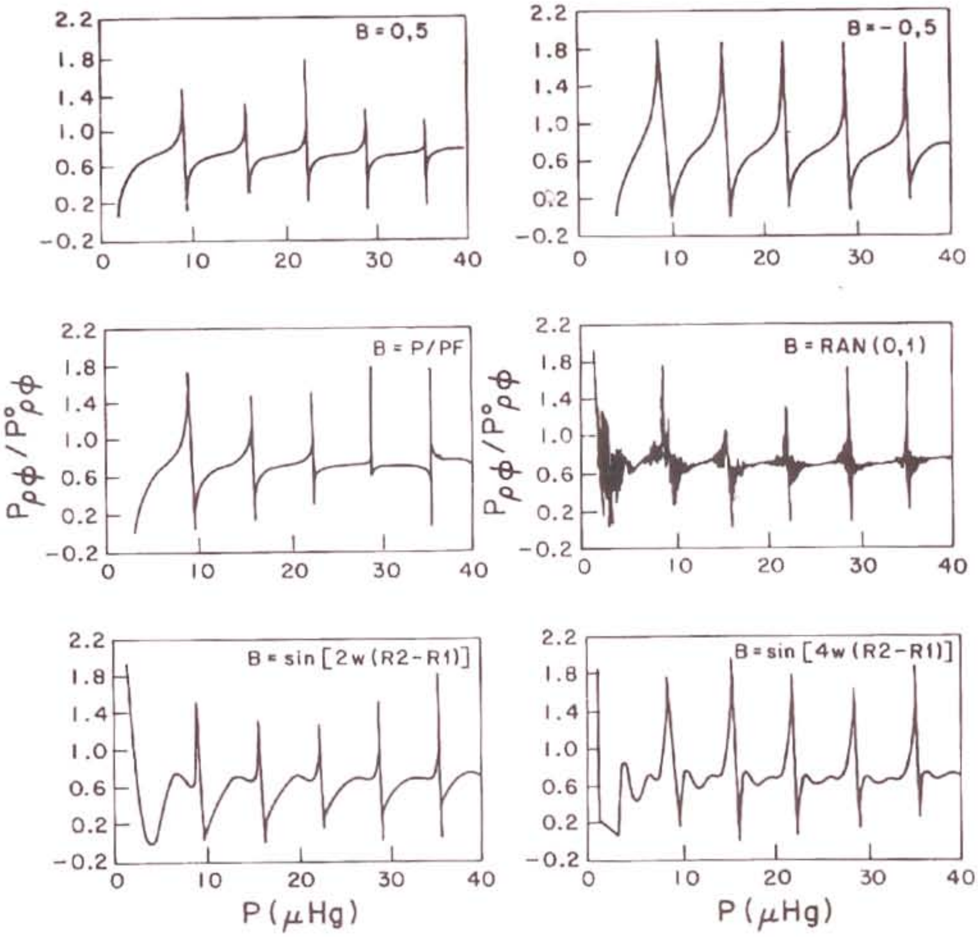

Figure 2. Effective viscosity $P_{\rho\phi}/P^{(0)}_{\rho\phi}$ determined from Eq. (24), vs preassure from p=0 to p=40 μHg, for a gas with cross-section $\sigma=80 \times 10^{-16}$ cm^2, T=293 K, R_1=2.5 cm and R_2=5.4 cm for B=0.5, -0.5, p/40, RAN(0,1), sin[$2\omega(R_2-R_1)$] and sin[$4\omega(R_2-R_1)$].

steady state of unaccelerated layers. For the planar case this condition of laminar fluid is introduced through the requirement that P_{zx} be independent of the z coordinate, and, in the present paper, through Equation [16], which is a condition of concentric unaccelerated layers (15).

IV. Concluding Remarks

Following a simple procedure derived from the path integral method we have obtained an expression for the effective viscosity of a gas in the transition regime. The gas has been considered to be confined between two concentric cylindrical surfaces given rise to an expression for the effective viscosity more general than that obtained previously for a planar geometry. We have found that the curvature has a moderate effect on viscosity in the condition analyzed in the present paper. In particular, the spatial variation of density produced by rotation has been neglected, a situation frequently occurring in coaxial cylinders systems used in experimental measurements of viscosity. For the case of cylindrical geometry analyzed here a pattern of rapid variations of the effective viscosity within equally spaced narrow regions in density is obtained in the transition regime, an analogous result to that previously obtained for the planar geometry case. Therefore, according to the results of this paper, the regions of rapid variation of the effective viscosity with density in the transition regime predicted by our theoretical expressions seem not to be a consequence of the rather artificial geometry of a system formed by a gas between infinite parallel plates (4).

Finally, we have examined how the dependence of the effective viscosity with density in the transition regime is perturbed when variation of the boundary conditions with density is considered. We have found that monotonic and non monotonic (oscillating and random) variations of B (K_n) with K_n does not destroy the pattern of rapid variation of the effective viscosity within equally spaced narrow regions of K_n.

Acknowledgment

One of us (MGS) acknowledges financial support from CNPq (Brazilian Agency), and Oscar Aurenty for technical assistance.

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