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Wave propagation in magnetic media

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Abstract

The purpose of this work is to investigate on the phenomenon of front propagation into magnetic media. Here, we study the case when the magnetization, M, is driven by a dc applied magnetic field, H_0, from the demagnetized to the magnetized state. A theoretical model is presented for solving the Landau-Lifshitz-Gilbert equation (LLGE) in the framework of an effective field that includes first order cubic, H_1, inplane uniaxial, H_U, and shape anisotropy fields, H_D. It is show that the dynamics of the magnetization is govern by a diffusion-reaction equation, and in the important case of uniformly translating profiles, this equation gives a family of solutions that describe harmonic oscillating (HO), damped oscillating (DO), exponential fronts (EF), amplified oscillating (AO), and dual front profiles (DF). Also of interest is the existence of a critical front speed, v^*, that separates the damped oscillations from the exponential fronts. In the case of purely uniaxial systems, this velocity is connected with the existence of a nonlinear marginal stability point for front propagation, and shows a strong dependence on the relative value of the anisotropy constants of the medium. When the crystalline anisotropy overcomes the uniaxial field the marginal stability point is not well defined since v^* is purely imaginary and only exponential fronts are linearly stable.

Key words: Fronts, magnetic media; magnetic waves; Landau-Lifshitz-Gilbert equation.

Propagación de ondas en medios magnéticos

Resumen

El propósito de este trabajo es estudiar el fenómeno de propagación de frentes de onda en medios magnéticos. Estudiamos el caso cuando la magnetización, M, evoluciona por la acción de un campo magnético dc, H_0, desde el estado desmagnetizado al estado magnetizado. Presentamos un modelo teórico para resolver la ecuación de Landau-Lifshitz-Gilbert (LLG) considerando un campo efectivo que incluye las anisotropias cúbica de primer orden, H_1, uniaxial en el plano, H_U, y de forma o desmagnetizante, H_D. Se muestra que la dinámica de la magnetización está gobernada por una ecuación de reacción-difusión y, en el importante caso perfiles uniformes, se obtiene una familia de soluciones que describen oscilaciones armónicas (HO), oscilacio-

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nes amortiguadas (DO), frentes exponenciales (FE), oscilaciones amplificadas (AO) y frentes duales (DF). Es interesante la existencia de una velocidad de frente crítica, υ^*, que separa las oscilaciones amortiguadas de los frentes exponenciales. En el caso de un sistema uniaxial puro, esta velocidad está relacionada con la existencia de un punto de estabilidad marginal no lineal, con una fuerte dependencia en los valores relativos de las constantes de anisotropia del medio. Cuando la anisotropia cristalina sobrepasa el campo uniaxial, el punto de estabilidad marginal no está bien definido, ya que υ^* es imaginario puro y sólo los frentes exponenciales son linealmente estables.

Palabras clave: Ecuación de Landau-Lifsshitz-Gilbert; medios magnéticos; ondas magnéticas.

Introduction

One of the most exciting subjects on condensed matter physics is that related to wave propagation into magnetic materials, and recently, the dynamics of the magnetization process in magnetic systems has become an updated topic. Intuitively, if a magnet is initially demagnetized, and when an external field is applied, the demagnetized state becomes unstable and the material is magnetized up to saturation. During the magnetization rotation, and due to domain wall motion, magnetic pulses, fronts and other profiles may propagate through a magnetic medium with a well defined dynamics, which leads to various nonlinear phenomena. These fronts are of many different types: solitons, shocks, dissipative fronts, uniformly translating profiles, and appear as solutions of a kind of nonlinear partial differential equations called in the literature as diffusion-reaction equations (DRE) (1). Numerical studies involving diffusion-reaction equations indicate that the velocity of the profile of these solutions is selected via some common dynamical mechanism, and that this selection mechanism is always related to marginal stability (2).

On the other hand, a magnetic material exhibit a nonlinear relation between the magnetization and the applied magnetic field that affect not only the switching process, but also distorts magnetic pulses that propagate through the medium. Since magnetic materials are commonly used for technical and practical applications, it is clear that the study of nonlinearities in these systems is of great importance.

In this paper front propagation in a magnetic system with uniaxial and cubic crystal anisotropies is considered. Based on the Landau-Lifshitz-Gilbert equation (LLGE) (3), we develop a model for a magnetic system in the presence of an external magnetic field, and show that in this framework the dynamics of the magnetization is governed by a nonlinear diffusion-reaction equation. The model is then solved numerically and the results interpreted in terms of concepts related with fronts propagating into an unstable state. It is also found that the velocity of the magnetic front is determined by a selection mechanism mainly due to the magnetic anisotropies of the medium and the applied field. Stability maps for uniformly translating profiles through magnetic media are proposed.

The paper is organized as follows. In section 2 we derive the nonlinear partial differential equation that govern the dynamic of the magnetization in the magnetic material with uniaxial and cubic anisotropies. In section 3 physical picture of front propagation is discussed, and we linearize the diffusion-reaction equation resulting from the analysis presented in section 2, and study the asymptotic solutions. Section 4 is devoted to the numerical analysis of the ordinary differential equation obtained in Section 2, and discuss the main results. In section 5 we summarize the main results from this study.

Theoretical approach

We start by assuming that the dynamics of the magnetization in the specimen is governed by the Landau-Lifschitz-Gilbert equation (LLGE) (3),

$$\frac{\partial}{\partial t}\vec{M} = -\gamma\vec{M}\times\vec{H} - \lambda\vec{M}\times\vec{M}\times\vec{H} \qquad [1]$$

where γ and λ, are the gyromagnetic ratio and the Gilbert damping parameter, respectively, M is the magnetization, $H = -\nabla_M E$, is the effective magnetic field, and E is the magnetic free energy. The first term on the right-hand side represents the precessional torque exerted by the magnetic field and the second term is a damping torque due to viscous forces acting on the magnetization. For the magnetic energy we used the Landau-Ginsburg free energy expanded in powers of the magnetization near a critical point, and may be expressed as

$$E = -\vec{H}_0\cdot\vec{M} + \sum_i \frac{(K_{ui} - D_i)}{M^2} M_i^2 + \sum_{ij}^{i\neq} \frac{K_{li}}{M^4}\left(M_i^2 M_j^2\right) + \frac{J}{M}\left(\nabla\vec{M}\right)^2 \quad (i=x,y,z) \qquad [2]$$

The first term in the right-hand side is the Zeeman energy, the second term represents the contribution from the uniaxial and demagnetizing anisotropies, the third term is the first order cubic anisotropy energy, and the forth term the exchange energy, with K_{ui}, D_i, K_1, and J denoting the uniaxial anisotropy, demagnetizing, crystalline, and exchange constants, respectively. H_0 is an external dc applied field. With the help of expression [2] and the definition of the effective field, [1] transforms into the 3 × 3 system,

$$M_x = -\gamma(M_y H_z - M_z H_y) - \lambda[M_y(M_x H_y - M_y H_x) - M_z(M_z H_x - M_x H_z)] \qquad [3a]$$

$$M_y = -\gamma(M_z H_x - M_x H_z) - \lambda[M_z(M_y H_z - M_z H_y) - M_x(M_x H_z - M_z H_x)] \qquad [3b]$$

$$M_z = -\gamma(M_x H_y - M_y H_x) - \lambda[M_x(M_z H_x - M_x H_z) - M_y(M_y H_z - M_z H_y)] \qquad [3c]$$

where the effective field components are given by

$$H_i = H_{0i} - \frac{2}{M^2}(K_{li} + K_{ui} - D_i)M_i + \frac{2K_{li}}{M^4}M_i^3 - \frac{J}{M^2}\nabla^2 M_i \quad (i=x,y,z) \qquad [4]$$

For sake of simplicity, we will consider a FM of length, L, large compared to it width, w ($w \ll L$), and the thickness, t, sufficiently small ($t \ll w$), so $\nabla \to \partial / \partial x$. The plane of the FM is in the (x, y) plane and an external dc magnetic field, H_0, is applied along the axis of the material (x-axis). It is also considered that the magnetization is completely inplane $\left(M_z^2 \ll M_x^2 + M_y^2 \approx M^2\right)$, the value of the cubic anisotropy is constant along all crystalline axes, and both demagnetizing and uniaxial fields are parallel to the x-axis.

With all these assumptions, [3a] to [3c] are rewritten as

$$M_x = \lambda\left(M_x H_y - M_y H_x\right)M_y \qquad [5a]$$

$$M_y = -\lambda\left(M_y H_z - M_z H_y\right)M_z \qquad [5b]$$

$$M_z = \gamma\left(M_y H_x - M_x H_y\right) \qquad [5c]$$

with the effective field components

$$H_x = H_{0x} - \frac{2}{M^2}(K_1 + K_u - D)M_x + \frac{2K_1}{M^4}M_x^3 - \frac{J}{M^2}\frac{\partial^2}{\partial x^2}M_x \qquad [6a]$$

$$H_y = -\frac{2K_1}{M^2}M_y + \frac{2K_1}{M^4}M_y^3 - \frac{J}{M^2}\frac{\partial^2}{\partial y^2}M_y \qquad [6b]$$

In case of a damping sufficiently high the torque part can be neglected, so $\lambda \gg \gamma$ and $M_z \approx 0$. This reduces our problem to solving [5a] together with the relation $M^2 = M_x^2(x,t) + M_y^2(x,t) = const$. Substitution of [6a], [6b] into [5a], lead to the nonlinear partial differential equation for the magnetization

$$\varphi(x,t) = a\frac{\partial^2}{\partial x^2}\varphi(x,t) + a\frac{\varphi(x,t)}{1\varphi^2(x,t)}\left(\frac{\partial}{\partial x}\varphi(x,t)\right)^2$$
$$+c_5\varphi^5(x,t) - c_3\varphi^3(x,t) + c_0.\varphi^2(x,t) + c_1\varphi(x,t) - c_0 \quad [7]$$

where $(x,t) = M_x(x,t)/M$, $H_u = 2K_u/M$, $H_D = 2D/M$, $H_E = J/M$, $a = H_E M$, $c_0 = H_0 M$, $c_1 = (H_u - H_D)M$, $c_3 = (H_u - H_D)M$ and $c_5 = 2H_1 M$. Within the mean-field approximation, the only important configurations near the critical point are those of uniform magnetization density, i.e. $\partial\varphi/\partial x = 0$. This brings [7] to the diffusion-reaction equation,

$$\varphi(x,t) = a\frac{\partial^2}{\partial x^2}\varphi(x,t) + c_5\varphi^5(x,t)$$
$$-c_3\varphi^3(x,t) + c_0\varphi^2(x,t) + c_1\varphi(x,t) - c_0 \quad [8]$$

where $c_2 = c_0 + a$.

Front propagation and linear stability

A special class of solutions of [10] are those called fronts. Fronts are extremely important in physics and are mathematically described by

$$\varphi(x,t) = \varphi(\xi = x - vt) \quad [9]$$

These solutions connect the unstable state $\mathrm{Lim}\varphi(\xi)$= 0 (demagnetized) to the stable state $\mathrm{Lim}\varphi(\xi)=\leq 1$ (magnetized), or

$$\lim_{\xi\to-\infty}\varphi(\xi) = 0 \quad [10]$$

$$\lim_{\xi\to-\infty}\varphi(\xi) = \varphi_s \quad [11]$$

Substitution of conditions [10] and [11] into [8] give the ordinary non-linear equation

$$\varphi''(\xi) = \frac{v}{a}\varphi'(\xi) + \frac{c_5}{a}\varphi^5(\xi) - \frac{c_3}{a}\varphi^3(\xi)$$
$$+\frac{c_0}{a}\varphi^2(\xi) + \frac{c_1}{a}\varphi(\xi) - \frac{c_0}{a} \quad [12]$$

The stable solutions of [14] are given by:

$$\frac{H_0}{H_E}(1-\varphi^2) + \left(\frac{H_1 + H_U - H_D}{H_E}\right)\mathrm{x}$$
$$\varphi - \left(\frac{3H_1 + H_U - H_D}{H_E}\right)\varphi^3 + \frac{H_1}{H_E}\varphi^5 = 0 \quad [13]$$

This equation gives five roots for each value of H_0/H_E. In the simple case of a pure uniaxial magnet ($H_1 = 0$), [8] reduces to the damped-forced Duffing's equation, and has been solved previously for an uniaxial magnet (4).

Apparently, the equation [12] has acceptable solutions for any value of v. However, according to other authors (5) there are some natural conditions for which dynamical velocity selection takes place, and then the velocity of all acceptable fronts converge asymptotically to one particular value. To study the stability of fronts, let us perturb the magnetized state (stable solution) with a small term $\delta\varphi$, such that, $\varphi \to \varphi_s - \delta\varphi$ as $\xi \to \infty$, where $|\delta\varphi| \ll \varphi_S$. Introducing the solution $\varphi = \varphi_S - \delta\varphi$ into [15], and retaining only terms up to first order in $\delta\varphi$, we obtain the linear equation

$$\frac{d^2}{d\xi^2}\delta\varphi + \frac{v}{a}\frac{d}{d\xi}\delta\varphi + \varepsilon\delta\varphi = 0 \quad [14]$$

where

$$\varepsilon = \frac{-5c_5 + 3c_3 - 2c_0 - c_1}{a} = \frac{2(H_U - H_D - H_1 - H_0)}{H_E} \quad [15]$$

with the corresponding characteristic polynomial

$$\lambda^2 + \frac{v}{a} + \varepsilon = 0 \quad [16]$$

The roots of this polynomial are given by

$$\lambda_{\pm} = -\frac{v}{2a} \pm \sqrt{\left(\frac{v}{2a}\right)^2 - \left(\frac{v^*}{2a}\right)^2} \quad [17]$$

where we have defined $v^* = 2a\varepsilon^{1/2}$. This yields to the following solutions

$$\delta\varphi = e^{-\frac{v}{2a}\xi}\left(Ae^{-\sqrt{v^2-v^{*2}}\frac{\xi}{2a}} + Be^{\sqrt{v^2-v^{*2}}\frac{\xi}{2a}}\right) \qquad [18]$$

Since we require that $\delta\varphi \to 0$ as $\xi \to \infty$, we get the asymptotic form $\delta\varphi \propto E^{-K\xi}$, whit

$$K = \frac{v}{2a} + \sqrt{\left(\frac{v}{2a}\right)^2\left(\frac{v^*}{2a}\right)^2} \qquad [19]$$

or, equivalently,

$$\frac{v}{a} = K + \frac{\varepsilon}{K} \qquad [20]$$

This expression gives the branch for asymptotic behavior, and is plotted in Figure 1 for $e = 0.1$ and 0.2. Every front profile in this branch is purely exponential and propagate with velocity $v \geq v^* = 2a\varepsilon^{1/2}$. This region is characterized by a continuum of attractors for any value of v, and is governed by a nonlinear-marginal-stability scenario, being the speed $v = v^*$ a transition point of marginal stability (3). In the case when $v < v^*$, $K = 1/2a\,[v + i(v^{*2} - v^2)^{1/2}]$, and the front oscillates with "natural frequency" $v^*/2a = \varepsilon^{1/2}$, and damping parameter $v^*/2a$. These results are summarized in Figura. 2, where the relation $v^* = 2a\varepsilon^{1/2}$ is plotted with respect to H_0/H_E. This map differentiates the oscillating fronts from the exponential front profiles. Each point on this curve correspond to a minimum in [20] for each value of H_0/H_E and $(H_U\text{-}H_D\text{-}H_1)/H_E > H_0/H_E$. When $H_U\text{-}H_D\text{-}H_1/H_E < H_0/H_E$, the marginal stability transition is not well defined since v^* is imaginary and the only stable states under perturbations are purely exponential front profiles.

Numerical Analysis

In order to understand the problem of wave propagation through magnetic media, we must go back to [14]. By means of a Runge-Kutta-Fehlberg scheme (6) this equation is solved numerically on a magnet of normalized length ξ/L with the boundary conditions [12] and [13], for several values of v, magnetic fields in the range $0 \leq H_0 \leq H_E$, and $0 \leq H_U \leq H_D - H_1 \leq H_E$. We obtain a family of solutions that describe harmonic oscillations (HO), damped oscillations (DO), exponential front profiles (EF), amplified oscillations (AO), or dual front profiles (DF), depending on the relative value of $(H_U\text{-}H_D\text{-}H_1)/H_E$, and on the wave speed, v. The main results obtained from these

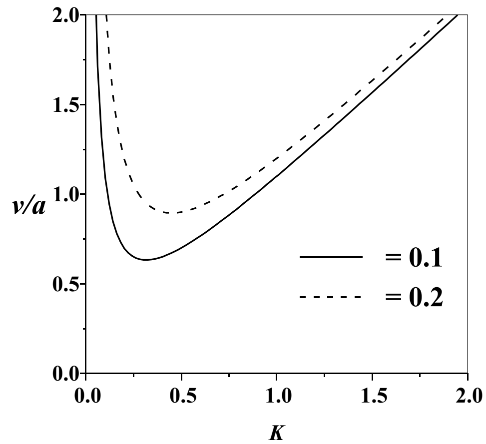

Figure 1. Branch of asymptotic behavior for front profiles as given by equation [20].

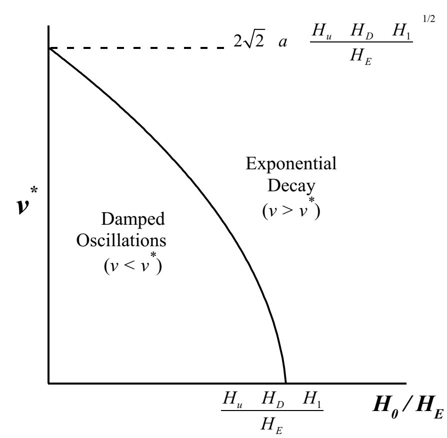

Figure 2. Dependence of v^* with respect to the applied magnetic field, H_0, for a representative value of $(H_U^- H_D^- H_1/H_E)$.

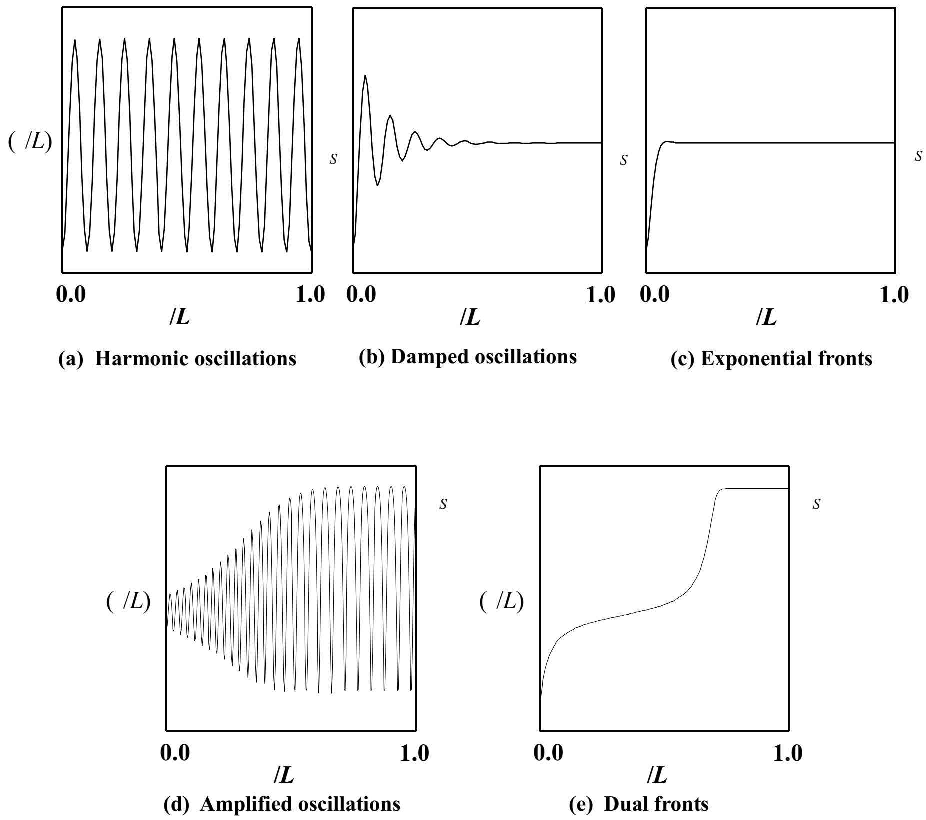

Figure 3. Representative solutions of equation [18] on a magnetic material of normalized length ξ/L: (a) harmonic oscillations, (b) damped oscillations, (c) exponential fronts, (d) amplified oscillations and (e) dual fronts.

calculations are summarized in Figure 3: (a) harmonic oscillations, (b) damped oscillations, (c) exponentials fronts, (d) amplified oscillations, and (e) dual fronts. We found a critical velocity, $\upsilon = \upsilon_0(H_0)$, at which only metastable harmonic oscillations are possible. In uniaxial magnets (H_U -$H_D > H_1$) no stable patterns exist for $\upsilon < \upsilon_0(H_0)$. For front speeds greater than $\upsilon_0(H_0)$, *M* stabilize around the state φ_S ($\leq$ 1), through damped oscillations up to the asymptotic value $\upsilon^*(H_0)$, in which the magnetized state invades rapidly the demagnetized state and an exponential front begins to propagate. The dashed line in Figura3(a) correspond to the value of φ_S around which the magnetization converge asymptotically. When H_U -H_D $< H_1$ all fronts represented in Figure 3 can propagate. The amplified oscillations are seen only for $\upsilon < \upsilon_0$ and in a narrow range of velocities. Usually related to parametric resonance conditions, amplification of the amplitude of a propagating pulse has been found experimentally and verified theoretically in systems such as magnetized coaxial transmission lines (7), thin ferromagnetic films (8), and microstrip antennas (9). Profiles of the type illustrated in Figure 5(e) are known as dual fronts (10, 11). In the presence of an external magnetic field such patterns develop under the special conditions ($K_1 > K_u$). This phenomenon occurs when the interface between an unstable state and a stable state splits into a two-fold interface developing two fronts propagating with different speeds.

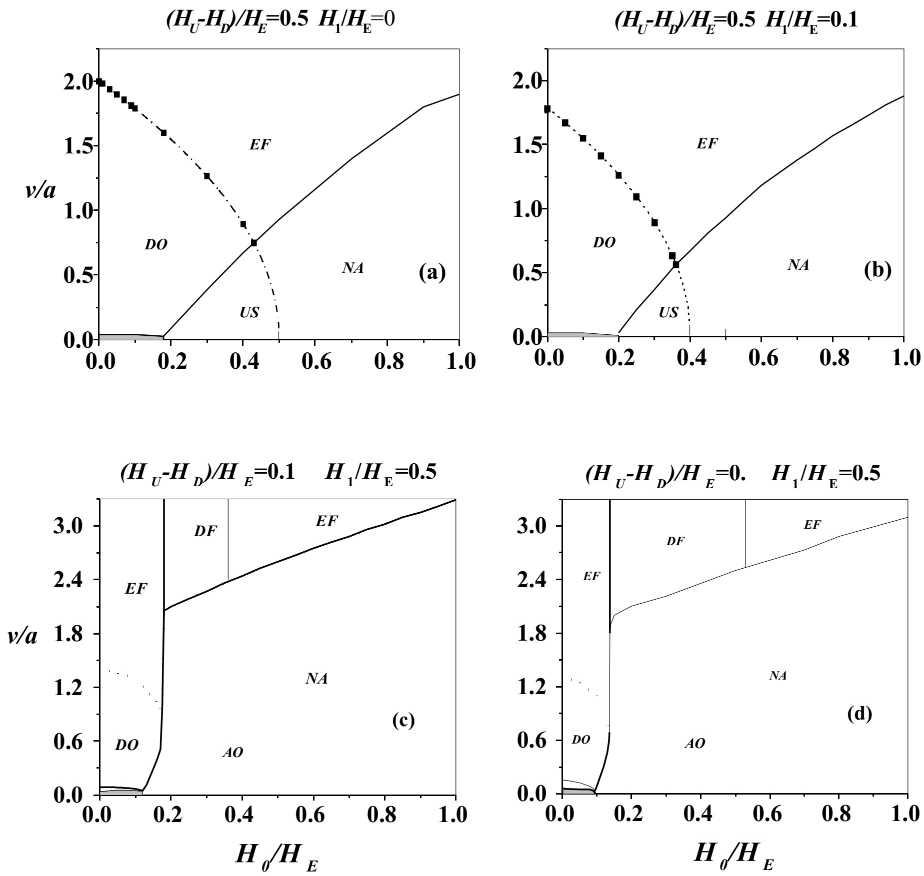

Figure 4. Stability diagrams for front propagation in a magnetic material, as obtained from solving numerically equation [18].

The stability maps ($\upsilon(H_0)$) for a magnetic medium are summarized in Figura 4, for (a) $H_1 = 0$, (b) $H_U - H_D > H_1 \neq 0$, (c) $H_U - H_D < H_1$, and (d) $H_U - H_D = 0$, as a function of the dc magnetic field. The closed squares in Figures 4(a) and 4(b), indicate the velocities observed in the stable solutions of [14]. The dashed curve represents the asymptotic value determined by the relation $\upsilon^*(H_0)$. The regions labeled DO and EF refer to damped oscillations and exponential profiles respectively, and are strongly dependent on the value of $(H_U\text{-}H_D\text{-}H_1)\ /H_E$. There also exist a region in which all states are unstable (US) and grows as $(H_U\text{-}H_D\text{-}H_1)\ /H_E$ increase. A small region of unstability is also observed with fronts propagating with velocity $\upsilon < \upsilon_0$ (shadowed). The region labeled as NA is physically inadmissible since $\varphi_S > 1$. For $H_U - H_D < H_1$ and $H_U - H_D = 0$, the situation is quite different, as depicted in Figures 4(c) and 4(d). Three features are characteristic of these maps: (a) the transition velocities from DO states to EF states do not correspond to a transition point of marginal stability, since when $H_U - H_D - H_1 \leq H_0 /H_E$ the values of $\upsilon^*(H_0)$ are purely imaginary, (b) a small region of amplified oscillations (AO) is observed for velocities below υ_0 between the DO and US regions, and (c) the region in which dual fronts are observed appear inserted between two regions of exponential profiles. It is also observed that the regions of AO and DF profiles grow as H_1 is increase.

Concluding remarks

In conclusion, we have shown that the dynamics of the magnetization in a magnetic material with uniaxial and magnetocrystalline anisotropy is governed by a nonlinear diffusion-reaction equation. In the case of uniform translating profiles it is found that the velocity of the profiles is determined by a selection mechanism mainly due to the magnetic anisotropies of the material. When the initially demagnetized system is perturbed by a dc applied field, profiles such as harmonic oscillations, damped oscillations, exponential fronts, amplified fronts and dual fronts can propagate through the medium, depending on the value of the applied field and the internal anisotropy fields. This model allowed us to propose stability diagrams for front propagation in a magnetic medium.

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